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VENTURE FINANCING OF START-UPS: A MODEL OF CONTRACT BETWEEN VC FUND AND ENTREPRENEUR

ABSTRACT: *Venture capital has become one of the main sources of innovation in the modern, global economy. It is not just a substitute for bank loans: it has proven to be a more efficient way of financing projects at different stages. On one hand, venture financing allows for projects with higher risk, which leads to the possibility of higher returns on investment. On the other hand, venture investors who usually have managerial experience often participate in governing the business, which certainly adds value to the enterprise. In this paper we establish the model of contract between the venture capital fund and the entrepreneur, focusing on probably the most important issue of this contract: the shares of the parties in the business. The shares in the company determine the distribution of the joint surplus. The expected joint profits are not just exogenously specified in the contract but are dependent on the behavioural variables of both parties at the stage of fulfilling the contract. We call the behavioural variable of the entrepreneur 'effort' and the one of the venture fund 'advice'. The probability of the project's success, and hence the expected joint revenues, are increased by these two. However, both kinds of effort are costly to the respective parties that have made them. Based on this fact we can elaborate the profit functions of both sides of the contract. Our model can be considered as a basis for specifying contracts concerning venture financing. It can provide the logic for how the equilibrium shares of entrepreneur and venture fund are obtained.*

KEY WORDS: *venture capital, entrepreneur, venture fund, value added, contract shares, costly effort, profit functions*

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1. INTRODUCTION

Venture financing is a well explored development which came into existence more than half a century ago. However, empirical research and modelling the processes related to venture capital emerged only within the last two decades. The basis of venture financing is the contract between the financing side (it could be a business angel, limited partnership, limited liability company or a fund structure; for simplicity we will call it venture capital fund, or VC) and the financed side (entrepreneur).

There are several aspects of venture financing which draw researchers' attention. The first of them is the influence of fiscal authorities on the volume of venture financing as well as on the incentives of both sides of the contract. Keuschnigg (2002) and Keuschnigg and Nielsen (2003a, b) analyze the implications of fiscal policy on venture-backed entrepreneurship. Gordon (1998) and Cullen and Gordon (2002) demonstrate that at a certain stage taxation may crowd out entrepreneurship (alternative earnings such as wages may become more preferable) which may lead to the disappearance of venture financing. Keuschnigg (2003) derives the before-tax equilibrium from the intersection of the demand and supply of innovative products and also assesses the social welfare after the deal has been executed. Further the author considers various components of fiscal policy (different types of taxes and subsidies) and also the policy that combines some of these components. Keuschnigg (2005) presents the rich approach of fiscal regulation of venture financing. The author discusses not only such fiscal mechanisms as research grants, entry subsidy for venture capitalists, and output subsidy (explored in his earlier papers), but also includes in his analysis such instruments as wage tax, cost of capital subsidy, capital gains tax, subsidies to new firms, and corporate tax.

Another aspect of venture financing that has recently been explored is the double-sided moral hazard problem in contractual relationship between the entrepreneur and the VC (Verkasalo (2006), Keuschnigg and Nielsen (2007), Schmidt (2003), Keuschnigg (2003) and others). Keuschnigg and Nielsen (2007) state that moral hazard is related to another phenomenon, adverse selection caused by the imperfection of market information. In their paper the authors demonstrate that revertible contracts may help to deal with moral hazard and hence to secure a better choice of behaviour by the two parties. If in the contract there exists a point cancelling the contract on condition of misconduct of any party; it will not only be a stimulus to effective business, but also will help to avoid dealing with 'bad' entrepreneurs and/or 'bad' projects. In order to reveal and estimate the problem of moral hazard Keuschnigg and Nielsen introduce a variable which acts as a signal of the quality of a given business project. The moral hazard problem is in suboptimal (smaller than those which maximize total surplus) levels of effort from both the entrepreneur and the VC. This problem results in the decrease of a project's success probability, and hence to welfare losses. Schmidt (2003) sees the reason for the VC's underinvestment in the following: all the financial investments are on behalf of the VC; however, by the end of the project the total surplus is shared (not fully absorbed by the financing side), which results in the VC's lower incentive to invest. The following framework is similar to Williamson's Transaction cost-property rights approach. The reason for moral hazard on behalf of the entrepreneur is opportunism: since the entrepreneur's effort is unobservable for a certain period of time the latter can choose a lower level of effort and hence his costs will decline. Keuschnigg (2003) states that existence of quality projects in the market can be secured through active research of the market for innovative products. Such research will help to distinguish potentially successful projects from the mass and also promote faster growth of the innovation stream. For introducing the phenomenon of this type of market research Keuschnigg assumes that the whole population is divided into managers and researchers.

There has been a lot of empirical research of venture financing which demonstrates the importance of the phenomenon. Gompers and Lerner (2001) write that almost half of the US companies sold on IPO were financed by venture capital. The same authors claim in the 1998 paper that liberalization of investment from pension funds results in expansion of venture capital and growth of innovative industries. This once again proves that pension funds are one of the main venture investors in the USA. Kortum and Lerner (2000) in their research of the US innovative sector show that a disproportionally large share of innovation is a product of venture financing. Moreover, the authors stress that the highest intensity of the innovative stream of venture capital financing can be observed in the case of financing of projects in their earliest stages. Keuschnigg (2003) states that if an innovative idea is financed by VC then the probability of the product's success triples. Kortum and Lerner (2000) also show that the share of venture financing in total R&D expenditure (given the unchanged potential of this type of spending) grew steadily from 3% in 1983 to 14% in 1998.

Note once again that the basis of the relationship between VC and entrepreneur is a contract. The terms written in the contract depend on which side has greater bargaining power. Many authors discuss in their papers who imposes the demand, the entrepreneur on the financing side, or the VC on one or several entrepreneurs and their projects. In most cases the authors attribute the power to the VC, explaining this by the fact that any given VC can deal with many projects, and therefore an entrepreneur should accept the conditions dictated by the VC as given (Keuschnigg and Nielsen (2002)).

One of the most important features of any contract is its optimality for both sides. In our case there exist several approaches to optimality (several parameters to regulate in order to achieve the optimal solution). Kanniainen and Keuschnigg (2004) consider optimality from the VC's point of view: optimality is in the right choice of project portfolio at the first stage and choice of shares in the business at the second stage. Bergemann and Hege (1997) in their model explore the dynamics of the optimal shares of the VC and entrepreneur, and demonstrate that the entrepreneur's share declines through time in all cases.

Our model is also mostly concentrated on the optimality of the contract: however the optimality will be determined through the agents' choice of behavioural parameters impacting the expected common profit, and also its distribution between the parties of the contact. These behavioural parameters will be the entrepreneur's level of effort ('correctness' of spending the invested money) and the level of advice of business angels (for simplicity assume the VC and the group of business angels is represented in the same notion).

Furthermore, if we know the optimal discrepancy between the two parameters we can find the equilibrium shares in the business and also the equilibrium profits of the parties. Such analysis cannot be observed in the previous research.

The aim of this paper is to explore the venture financing contract of a start-up (implying investment in a project at an early stage). In Section 2 we present a model of two-stage interaction between VC and entrepreneur, resulting in the determination of optimal shares in the business. In Section 3 we will present numerical examples of the problem's solution, and also explain the dynamics of the change of shares and profits due to changes in the agents' various behavioural parameters. Section 4 concludes.

2. THE MODEL

In our paradigm signing the contract between entrepreneur and VC is the final stage of the interaction between the two parties. The most important aspect of this contract is determination of the shares in the business. Let us assume that it is the VC that determines the shares, since it is the investing side and therefore can either approve of the entrepreneur's idea (invest) or hold back from investing. The shares in the enterprise are not determined spontaneously but are the consequence of economic interaction. We will divide our analysis into two stages: in the first stage the VC maximizes its profit with respect to its share in the business, in the second stage the entrepreneur accepts the resulting shares and project realization begins. A project's success is positively related to the level effort chosen by the entrepreneur and the level of advice chosen by the VC. At the same time both parties incur costs, both exogenous and resulting from the respective undertaken levels of effort. Consequently in the second stage of interaction the equilibrium levels of effort and advice are determined. Knowing these levels in the first stage the VC maximizes its expected profit, thereby choosing its optimal share in the business. As we can see we are dealing with an example of economic analysis built on the inverse induction technique.

Consider the expected revenues and costs of each party in detail. Let us start with the entrepreneur and see how his profit function is formed. An individual starts with a decision to become either an entrepreneur or a worker with some wage rate w; the individual makes this choice according to his preferences (degree of risk aversion, subjective estimates of certain levels of income). Next, after becoming an entrepreneur, the individual creates a business idea incurring some effort h (the entrepreneur uses his knowledge and ideas for elaborating the project plan). The cost of this effort is $c_{eff}(h)$, $c_{eff}' > 0, c_{eff}'' > 0$. When the creation of the project is finished the entrepreneur starts negotiations with the VC. In the case where the VC accepts the project, the contract is signed. By conditions of contract the VC pays the start-up cost of the firm M and also incurs the upfront payment k to the entrepreneur (initial compensation). Another important issue in the contract is revenue distribution, namely the shares of the VC and the entrepreneur in the business (s and $(1-s)$, respectively).

In the next stage the entrepreneur and VC independently and simultaneously decide on their respective levels of effort for maintenance of the project. The VC inputs a (a certain amount of managerial advice for conducting the business, which is equivalent to research, analysis, and interpretation of the relevant market information). The VC, therefore, incurs the cost $c_V(a)$, $c_V' > 0, c_V'' > 0$. The entrepreneur inputs e (the level of effort) and incurs cost $c_E(e)$, $c_E' > 0, c_E'' > 0$. Both types of effort contribute to the overall probability of the project's success $p(e,a)$ (the relationship between both types of effort and this probability is positive for any values of e and a). In the last stage the project is run and the revenue from the project is distributed between the parties.

Consequently, the VC's expected profit from the project is as follows:

$$\pi_V = p(e,a)sR - c_V(a) - (M+k), \tag{1}$$

where the revenue from the project is R with probability $p(e,a)$ and 0 with probability $(1-p(e,a))$.

The entrepreneur's expected profit is as follows:

$$\pi_E = p(e,a)(1-s)R + k - c_{eff}(h) - c_E(e) \tag{2}$$

We are dealing with the model of simultaneous choice of level of effort e by the entrepreneur and the level of advice a by the VC. Hence, in order to solve the problem we need to find the respective reaction functions. For an explicit solution we need to present the functions $p(e,a)$, $c_V(a)$ and $c_E(e)$ in explicit forms. By economic intuition all three functions increase in their arguments. In addition the probability function is concave from the origin, whereas the cost functions are convex from the origin (given that both arguments are non-negative). $p(e,a)$ is concave because it should asymptotically approach 1 as we infinitely increase e and a. Cost functions are convex due to the axiomatic concavity of the profit functions. Assume $p(e,a) = e^{\alpha} a^{\beta}$; $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. In this case the probability function has the proposed shape. In order for the probability to be distributed between 0 and 1 we normalize e and a in such a way that the maximum value for both of these variables is 1 (or 100%) and their minimum value is 0. Let

$$c_V(a) = a^{\gamma}, \text{ where } \gamma > 1, \text{ and } c_E(e) = e^{\delta}, \text{ where } \delta > 1. \tag{3}$$

The representation of the functions is simple; however the generality is not lost. The equations of reaction function can be derived from first order conditions for profit function maximization: we will consider only the case of contract between a single VC and a single entrepreneur.

Derive VC's reaction function:

$$\pi_V(a) = e^{\alpha} a^{\beta} sR - a^{\gamma} - (M + k) \tag{4}$$

$$\pi_V{}' = \beta e^{\alpha} a^{\beta-1} sR - \gamma a^{\gamma-1} = 0$$

$$a^{\gamma-1} = \frac{\beta e^{\alpha} sR}{\gamma} \cdot a^{\beta-1}$$

$$a^{\gamma-\beta} = \frac{\beta e^{\alpha} sR}{\gamma}$$

$$a(e) = (\frac{\beta e^{\alpha} sR}{\gamma})^{\frac{1}{\gamma-\beta}} \tag{5}$$

The obtained reaction function increases on its domain and is concave from the origin. To prove it find its first and second derivatives and compare them with 0:

$\frac{\partial a^{VC}}{\partial e} = (\frac{\beta sR}{\gamma})^{\frac{1}{\gamma-\beta}} \cdot \frac{\alpha}{\gamma-\beta} \cdot a^{\frac{\alpha+\beta-\gamma}{\gamma-\beta}} > 0$ on condition that all Greek letters assumptions hold.

$\frac{\partial^2 a^{VC}}{\partial e^2} = (\frac{\beta sR}{\gamma})^{\frac{1}{\gamma-\beta}} \cdot \frac{\alpha}{\gamma-\beta} \cdot \frac{\alpha+\beta-\gamma}{\gamma-\beta} \cdot a^{\frac{\alpha+2\beta-2\gamma}{\gamma-\beta}} < 0$, since $\alpha + \beta < 1, \gamma > 1$.

By the same logic the entrepreneur's reaction function can be obtained:

$$\pi_E = e^{\alpha} a^{\beta} (1-s)R + k - c_{eff}(h) - e^{\delta} \tag{6}$$

$$\pi_E' = \alpha e^{\alpha-1} a^{\beta} (1-s)R - \delta e^{\delta-1} = 0$$

$$e(a) = (\frac{a^{\beta}(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}} \tag{7}$$

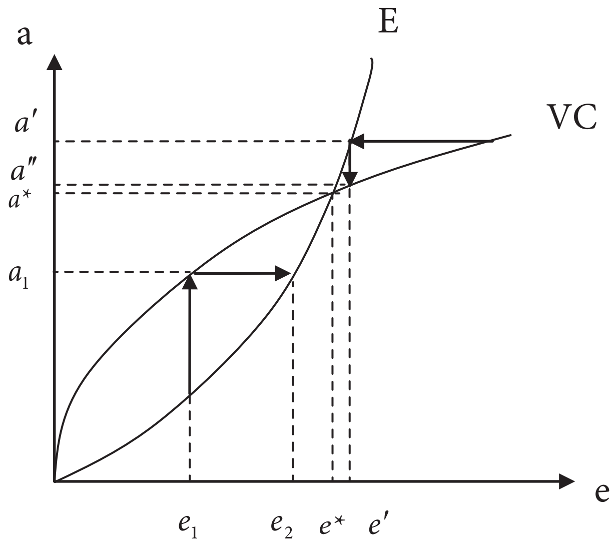
The entrepreneur's reaction function increases and is convex from the origin (in the same (e,a) framework):

$$\frac{\partial e^{E}}{\partial a} = (\frac{(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}} \cdot \frac{\beta}{\delta-\alpha} \cdot a^{\frac{\alpha+\beta-\delta}{\delta-\alpha}} > 0$$

$\frac{\partial e^{E}}{\partial a} = (\frac{(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}} \cdot \frac{\beta}{\delta-\alpha} \cdot \frac{\alpha+\beta-\delta}{\delta-\alpha} a^{\frac{2\alpha+\beta-2\delta}{\delta-\alpha}} < 0$, since $\alpha + \beta < 1, \delta > 1$

The obtained reaction functions can be depicted in (e, a) framework:

Picture 1. Sustainability of equilibrium in (e,a) framework.



From the described properties of the reaction functions it follows that the equilibrium exists (the graphs of the reaction functions intersect). It is worth mentioning that point (0,0) belongs to both graphs: however, we will not consider this intersection as equilibrium because, on one hand, it is irrelevant from the economic point of view, and on the other hand, this equilibrium is unsustainable.

Let us find the equilibrium in our system. Firstly, we will demonstrate in Picture 1 that point (e^*, a^*) (the point of intersection of our reaction functions) is a sustainable equilibrium. Assume that the entrepreneur initially chooses the level of effort that is smaller than e^* (say, e_1). Then the VC reacts by choosing the level of advice $a_1 = a(e_1)$ which maximizes its expected profit. In his turn the entrepreneur chooses the level of effort $e_2 = e(a_1)$. Consequently, after a number of such consecutive steps from both sides the equilibrium converges to point (e^*, a^*).

Now demonstrate that any deviation from (e^*, a^*) to the north-east will also end by returning to (e^*, a^*). To be clearer let us assume that this time it is the

VC who deviates from the equilibrium first (he chooses the level of advice a'). From the entrepreneur's reaction function it follows that he chooses the level of effort $e' = e(a')$, which results in the new choice of $a'' = a(e')$ by the VC, and so on until the equilibrium converges to (e^*, a^*).

We showed that any deviation from the reaction functions' intersection will result in step-by-step return to the initial equilibrium. We conclude that this equilibrium is steady (sustainable).

In order to move to the first stage of our game (the stage at which the VC maximizes its profit by its share in the business) we need to derive the equilibrium $(e(s), a(s))$ mathematically (see the full proof in the Appendix).

$$a(s) = \left[\frac{\left(\frac{\gamma}{\beta sR}\right)^{\frac{1}{\alpha}}}{\left(\frac{\alpha(1-s)R}{\delta}\right)^{\frac{1}{\delta-\alpha}}}\right]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}} = \left[\left(\frac{\gamma}{\beta R}\right)^{\frac{1}{\alpha}} \cdot \left(\frac{\alpha R}{\delta}\right)^{\frac{1}{\alpha-\delta}}\right]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}} \cdot s^{\frac{\delta-a}{\alpha\gamma+\beta\delta-\delta\gamma}} \cdot (1-s)^{\frac{\alpha}{\delta\gamma-\alpha\gamma-\beta\delta}} = \theta s^p (1-s)^q$$

where (8)

$$\theta = \left[\left(\frac{\gamma}{\beta R}\right)^{\frac{1}{\alpha}} \cdot \left(\frac{\alpha R}{\delta}\right)^{\frac{1}{\alpha-\delta}}\right]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}}, p = \frac{\alpha-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}, q = \frac{\alpha}{\delta\gamma-\alpha\gamma-\beta\delta}$$

$$e(s) = \left[\frac{\left(\frac{\beta sR}{\gamma}\right)^{\frac{1}{\gamma-\beta}}}{\left(\frac{\delta}{\alpha(1-s)R}\right)^{\frac{1}{\beta}}}\right]^{\frac{\beta(\gamma-\beta)}{\delta\gamma-\alpha\gamma-\beta\delta}} = \left[\left(\frac{\gamma}{\beta R}\right)^{\frac{1}{\beta-\gamma}} \cdot \left(\frac{\alpha R}{\delta}\right)^{\frac{1}{\beta}}\right]^{\frac{\beta(\beta-\gamma)}{\alpha\gamma+\beta\delta-\beta\gamma}} \cdot s^{\frac{\beta}{\delta\gamma-\alpha\gamma-\beta\delta}} \cdot (1-s)^{\frac{\gamma-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}} = \xi s^x (1-s)^y$$

(9)

where $\xi = \left[\left(\frac{\gamma}{\beta R}\right)^{\frac{1}{\beta-\gamma}} \cdot \left(\frac{\alpha R}{\delta}\right)^{\frac{1}{\beta}}\right]^{\frac{\beta(\beta-\gamma)}{\alpha\gamma+\beta\delta-\beta\gamma}}, x = \frac{\beta}{\delta\gamma-\alpha\gamma-\beta\delta}, y = \frac{\gamma-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}$

In order to determine the VC's optimal share in business, write down its expected profit as a function of its share, substituting the results from (8) and (9) into profit function (4):

$$\pi_V(s) = [\xi s^x (1-s)^y]^\alpha \cdot [\theta s^p (1-s)^q]^\beta sR - [\theta s^p (1-s)^q]^\gamma - (M+k) =$$
$$= \xi^\alpha \theta^\beta R s^{(\alpha x + \beta p + 1)} (1-s)^{(\alpha y + \beta q)} - [\theta s^p (1-s)^q]^\gamma - (M+k)$$

We will maximize the obtained expression under two constraints:

1) The entrepreneur's profit is no less than w (the threshold wage value), (incentive compatibility constraint).
2) The VC's profit is non-negative (individual rationality constraint).

The problem may be expressed in the full form as follows:

$$\xi^\alpha \theta^\beta R s^{(\alpha x + \beta p + 1)} (1-s)^{(\alpha y + \beta q)} - [\theta s^p (1-s)^q]^\gamma - (M+k) \xrightarrow[s]{} \max$$
$$s.t. \xi^\alpha \theta^\beta R s^{(\alpha x + \beta p)} (1-s)^{(\alpha y + \beta q + 1)} - [\xi s^x (1-s)^y]^\delta + k - c_{eff}(h) - w \geq 0,$$
$$\xi^\alpha \theta^\beta R s^{(\alpha x + \beta p + 1)} (1-s)^{(\alpha y + \beta q)} - [\theta s^p (1-s)^q]^\gamma - (M+k) \geq 0.$$

If we know the values of all parameters then it will be easy to solve this problem using the Lagrange multipliers method.

$$L(s, \lambda_1, \lambda_2) = \xi^\alpha \theta^\beta R s^{(\alpha x + \beta p + 1)} (1-s)^{(\alpha y + \beta q)} - [\theta s^p (1-s)^q]^\gamma - (M+k) +$$
$$+ \lambda_1 (\xi^\alpha \theta^\beta R s^{(\alpha x + \beta p)} (1-s)^{(\alpha y + \beta q + 1)} - [\xi s^x (1-s)^y]^\delta + k - c_{eff}(h) - w) +$$
$$+ \lambda_2 (\xi^\alpha \theta^\beta R s^{(\alpha x + \beta p + 1)} (1-s)^{(\alpha y + \beta q)} - [\theta s^p (1-s)^q]^\gamma - (M+k))$$

In order to maximize the obtained Lagrangian the following system should be solved:

$$\frac{\partial L}{\partial s} = (\lambda_2 + 1)[\xi^{\alpha}\theta^{\beta}Rs^{\alpha x+\beta p}(1-s)^{\alpha y+\beta q-1}((\alpha x+\beta p+1)(1-s)-(\alpha y+\beta q)s)-$$
$$-\theta^{\gamma}s^{\gamma p-1}(1-s)^{\gamma q-1}(\gamma p(1-s)-\gamma qs)]+$$
$$\lambda_1[\xi^{\alpha}\theta^{\beta}Rs^{\alpha x+\beta p-1}(1-s)^{\alpha y+\beta q}((\alpha x+\beta p)s+(\alpha y+\beta q+1)(1-s))-$$
$$-\xi^{\delta}s^{\delta x-1}(1-s)^{\delta y-1}(\delta x(1-s)-\delta ys)]=0$$
$$\lambda_1(\xi^{\alpha}\theta^{\beta}Rs^{(\alpha x+\beta p)}(1-s)^{(\alpha y+\beta q+1)}-[\xi s^{x}(1-s)^{y}]^{\delta}+k-c_{eff}(h)-w)=0$$
$$\lambda_1 \geq 0$$
$$\xi^{\alpha}\theta^{\beta}Rs^{(\alpha x+\beta p)}(1-s)^{(\alpha y+\beta q+1)}-[\xi s^{x}(1-s)^{y}]^{\delta}+k-c_{eff}(h)-w \geq 0$$
$$\lambda_2(\xi^{\alpha}\theta^{\beta}Rs^{(\alpha x+\beta p+1)}(1-s)^{(\alpha y+\beta q)}-[\theta s^{p}(1-s)^{q}]^{\gamma}-(M+k))=0$$
$$\lambda_2 \geq 0$$
$$\xi^{\alpha}\theta^{\beta}Rs^{(\alpha x+\beta p+1)}(1-s)^{(\alpha y+\beta q)}-[\theta s^{p}(1-s)^{q}]^{\gamma}-(M+k) \geq 0$$

Thus when signing the contract the entrepreneur will encounter the shares in the business proposed by the VC and will accept it if his expected profit is no less than the threshold wage. In the second stage (realization of the project), knowing their shares the entrepreneur and the VC will choose the optimal levels of effort and advice, respectively (equations (8) and (9)). We can say that we have found an algorithm for determining the equilibrium values of parameters *e*, *a* and *s*.

3. EXAMPLES

The problem presented above cannot be solved explicitly; therefore, let us consider several particular cases. For simplicity we will make the following initial assumptions:

Let R=1000, M=100, k=50, c(h)=25, w=70. Then we have four unknown parameters: $\alpha, \beta, \delta, \gamma$. By default, let these parameters equal 0.2, 0.2, 1.5, 1.5, respectively. Solve our problem given the values of all unknown parameters. The VC's maximization problem is as follows (copied from Maple):

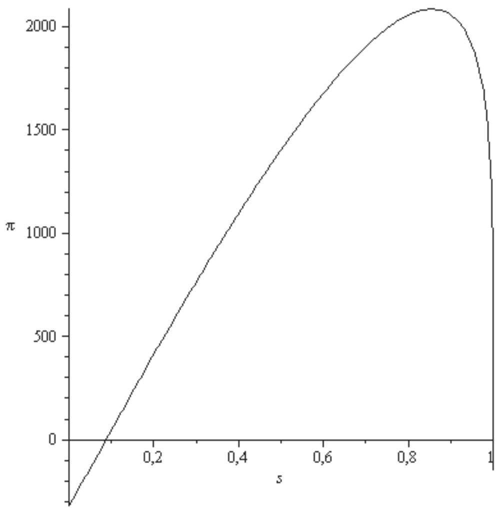
$$\mathit{maximize}\big(3929.962965\ s^{1.024242424}\cdot(1-s)^{0.1818181818} - 169.4168944\,(1-s)^{0.1818181818} - 150,\ s=0..1,\ \mathit{location}\big)$$

For further simplicity we will maximize the goal function first and check whether the inequalities hold at the point of maximum. The maximum value of the VC's profit turned out to be 2087.03 with its share equal to 0.856, which means that the condition that the VC's profit is non-negative holds. In order to check that the entrepreneur's constraint is satisfied write down this constraint and plug in the values of all parameters. The entrepreneur's expected profit is equal to:

$$3929.962965\ s^{0.024242424} \cdot (1-s)^{1.1818181818} - 169.4168944 \cdot s^{0.1818181818} \cdot (1-s)^{1.1818181818} + 25$$

If we put 0.856 instead of s at optimum the entrepreneur's profit will be equal to 405.402, which is greater than his reservation wage (70). Therefore, we can say that the problem is solved. Let us illustrate the profit of the VC as a function of its share in the business:

Picture 2. Graph of the relationship between the VC's profit and its share in the business ($\alpha = \beta = 0.2, \gamma = \delta = 1.5$)



Now let us see how the problem's solution changes if we change one of the parameters. Begin with $\alpha = 0.4$, ceteris paribus.

The maximization problem will be as follows:

$$maximize\big(18245.9513\, s^{1.088888889} \cdot (1-s)^{0.4444444445} - 2193.978804\, s^{0.2222222222}(1-s)^{0.4444444445} - 150, s = 0\,..1, location\big)$$

The maximum value of the VC's profit becomes 4971.156 and its new share is 0.734, which means that the constraint of non-negative VC's profit is satisfied, as in the initial problem.

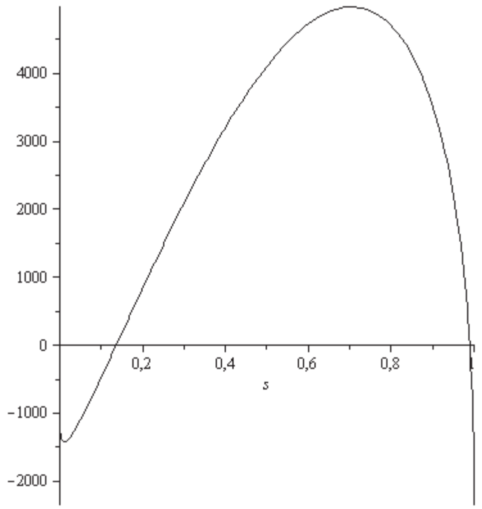
The entrepreneur's profit function is as follows:

$$18245.9513\, s^{0.088888889} \cdot (1-s)^{1.4444444445} - 2193.978804 \cdot s^{0.2222222222}(1-s)^{1.4444444445} + 25$$

At 0.734 the value of this function is 659.555, which is still greater than 70.

Graphical illustration:

Picture 3. Graph of the relationship between the VC's profit and its share in the business ($\alpha = 0.4, \beta = 0.2, \gamma = \delta = 1.5$).



We conclude that if ceteris paribus we increase the parameter α, the profits of both entrepreneur and VC increase and the VC's share in the business declines.

Consider another case. Now increase β to 0.4 keeping the other parameters of the initial problem unchanged. In this case the VC's maximization problem is as follows:

$$maximize\big(18245.9513 \cdot s \cdot (1-s)^{0.2222222222} - 1144.8753 \cdot s^{-0.2222222222}(1-s)^{0.2222222222} - 150, s = 0\,..1, location\big)$$

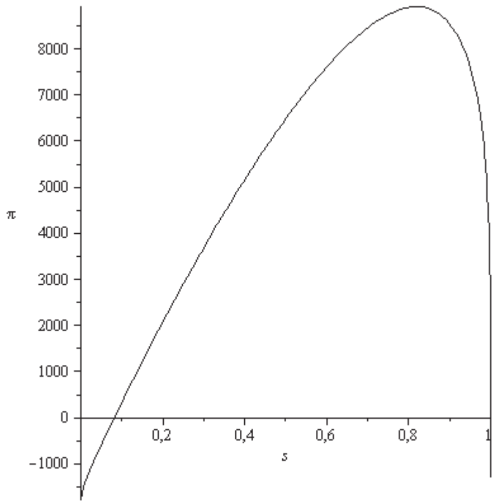
If we solve this problem we will obtain maximum profit of 9056.224 at the point s=0.826. The entrepreneur's expected profit function is as follows:

$$18245.9513 \cdot (1-s)^{1.2222222222} - 1144.8753 \cdot s^{0.4444444444}(1-s)^{1.2222222222} + 25$$

At s=0.826 the entrepreneur's expected profit is equal to 1145.573; hence, the entrepreneur's constraint is satisfied.

We conclude that if we increase parameter β leaving other parameters unchanged, the expected profits of both entrepreneur and VC increase: these increments are higher than the ones obtained from the growth in parameter α. The share of the VC in the business still falls.

Picture 4. Graph of the relationship between the VC's profit and its share in the business ($\alpha = 0.2, \beta = 0.4, \gamma = \delta = 1.5$).



Why in the case of growth in parameters α or β do both parties' profits increase and the VC's share in the business decline? Remember that both of these parameters directly affect the probability of the project's success. Expected revenues increase with α and β, but costs are not affected by the change in either α or β. Hence the profits of both parties increase with both of these parameters. At the same time the profits increase in higher proportions if we increment parameter β. This can be explained by the higher sensitivity of the VC's advice to β than to α. As a result the growth in beta brings about a higher increase in the equilibrium level of advice than in the same growth in alpha. In our case the level of the VC's advice has more influence on the total surplus than the level of the entrepreneur's advice, since, according to the assumptions of our model, it is the VC who participates in the first stage of the game (chooses the level of advice).

From equation (4) it follows that β (not α) is the behavioural parameter of the VC.

The VC's share in the business declines. We will prove it using equation (4) and the Implicit Function Theorem. Our goal is to find $\frac{\partial s}{\partial \alpha}$. Use IFT and express it through the level of advice:

$$\frac{\partial s}{\partial \alpha} = -\frac{\partial a / \partial \alpha}{\partial a / \partial s}$$

$$\left.\begin{array}{l} \dfrac{\partial a}{\partial \alpha} = -\dfrac{\partial \pi_V / \partial \alpha}{\partial \pi_V / \partial a} < 0 \\ \dfrac{\partial a}{\partial s} = -\dfrac{\partial \pi_V / \partial s}{\partial \pi_V / \partial a} < 0 \end{array}\right. \text{, since } \frac{\partial \pi_V}{\partial \alpha} > 0, \frac{\partial \pi_V}{\partial a} > 0, \frac{\partial \pi_V}{\partial s} > 0.$$

Hence, $\frac{\partial s}{\partial \alpha} < 0$. By analogy it can be proved that $\frac{\partial s}{\partial \beta} < 0$.

Now consider the growth of parameter γ to 2 holding other parameters of the initial problem constant. The VC's maximization problem is as follows:

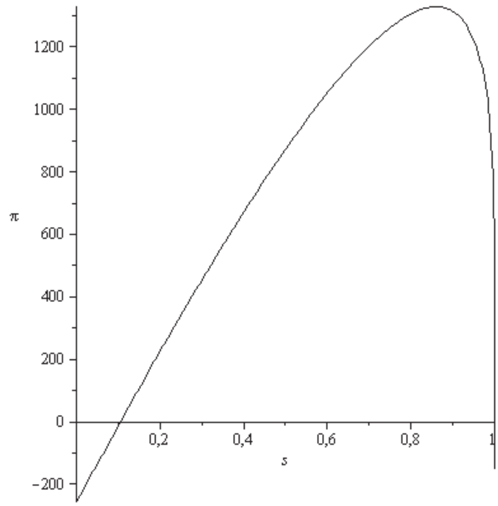
$$maximize\Big(2549.4193 \cdot s^{1.017391304} \cdot (1-s)^{0.1739130434} - 105.5369579 \cdot (1-s)^{0.1739139130435} - 150, s = 0..1, location\Big)$$

The solution is s=0.858 and max expected profit= 1328.27, which means that the VC's constraint is satisfied. The entrepreneur's profit function is as follows:

$$2549.4193 \cdot s^{0.017391304} \cdot (1-s)^{1.1739130434} - 105.5369579 \cdot s^{0.1304348726} (1-s)^{1.1739139130434} + 25$$

At the point of maximum s=0.858 the entrepreneur's expected profit is 179.034, hence the entrepreneur's constraint is also satisfied.

Picture 5. Graph of the relationship between the VC's profit and its share in the business ($\alpha = \beta = 0.2, \gamma = 2, \delta = 1.5$).



Compared to the default case the profits of both agents declined, whereas the shares in the business did not significantly change (the VC's share increased slightly).

Consider the last case ($\delta = 2$, other parameters of the initial problem are left unchanged). The VC's maximization problem is as follows:

$$maximize\left(2549.4193 \cdot s^{1.017391304} \cdot (1-s)^{0.1304347826} - 33.93755969 \cdot (1-s)^{0.1304347826} - 150, s = 0..1, location\right)$$

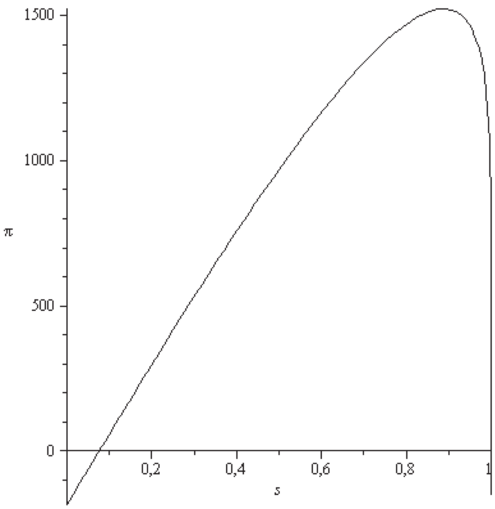
Maximum expected profit (1530.83) is obtained at the point s=0.885. The individual rationality constraint is satisfied.

The entrepreneur's expected profit function is as follows:

$$2549.4193 \cdot s^{0.017391304} \cdot (1-s)^{1.1304347826} - 33.93755969 \cdot s^{0.1739130435} (1-s)^{0.1304347826}$$

At the point s=0.885 the entrepreneur's expected profit is 186.804, which is higher than 70. The incentive compatibility constraint is satisfied. Compared to the initial case, the profits of both agents declined and the share of the VC increased.

Picture 6. Graph of the relationship between the VC's profit and its share in the business ($\alpha = \beta = 0.2, \gamma = 1.5, \delta = 2$).



Let us explain why the profits of both parties declined in the last two examples.

γ and δ are behavioural parameters of the VC and the entrepreneur, respectively. They demonstrate how a respective agent is averse to his own effort. Therefore, the greater is γ , the higher are the VC's costs at the same level of advice, which ceteris paribus means a decline in the VC's expected profit. As a result for the VC it becomes optimal to decrease the level of advice leading to the decline of the project's success probability, and hence of the entrepreneur's expected profit. By analogy, the entrepreneur's profit falls given the increase in δ leading to the decline in the entrepreneur's optimal level of effort and decrease in both success probability and the VC's expected profit. Note that in the case of growth in γ the decrease in both agents' profits is more significant than in the case of growth in δ . This happens because it is the VC who plays first in our two-stage game and hence has greater influence on common surplus. The VC's profit is more elastic with respect to the change in its own parameter (γ) rather than to change in the entrepreneur's behavioural parameter (δ).

Now let us explain why the effect of increasing γ in the VC's share is ambiguous, whereas there is a strong positive relationship between δ and *s*. Our explanation will contain elements of both mathematics and logic.

Rewrite $\frac{\partial s}{\partial \gamma}$ using equation (4) and the Implicit Function Theorem:

$$\frac{\partial s}{\partial \gamma} = -\frac{\partial \pi_V / \partial \gamma}{\partial \pi_V / \partial s}.$$

The denominator of the fraction is obviously positive. The numerator could be zero, positive, or negative. From equation (4) it directly follows that the numerator is positive ($\frac{\partial \pi_V}{\partial \gamma} = -a^{\gamma} \cdot \ln a$; the first multiple is negative as well as the second one, since $0 < a < 1$). However, we should understand that there exists an opposite effect. Ceteris paribus the growth in γ causes the VC's costs to increase. This leads to the choice of the lower amount of advice and to a decrease in the VC's expected profit. Which of the two effects dominates is unclear: everything depends on the parameters of the problem.

Consider the effect of δ on *s*.

$$\frac{\partial s}{\partial \delta} = -\frac{\partial \pi_V / \partial \delta}{\partial \pi_V / \partial s}.$$

The sign of the denominator is again positive. In this case, however, the sign of the denominator is unambiguous. Parameter δ is not explicitly present in equation (4), but has an implicit effect on the VC's expected profit. If δ increases the entrepreneur has an incentive to decrease the level of effort (because of the increase in his cost). The decrease of effort results in the decline of the project's probability of success and the VC's expected profit. As a result, the numerator is negative and $\frac{\partial s}{\partial \delta}$ is positive. Ceteris paribus the growth in parameter δ leads to the increase in the VC's share in the business.

4. CONCLUSION

We built a model of contract between an entrepreneur and a venture capital fund at the stage of head-to-head negotiations. In other words, we avoided the stage of choosing the right project from the total supply of innovative products. On one hand, this may seem a single-sided description of reality. On the other hand, it helped us to highlight the chosen stage of venture financing in greater detail. Our main contribution is in finding the optimal shares of the VC and the entrepreneur in a given contract. To find these shares we implemented a standard economic method of inverse induction. Hence, unlike many other models (Keuschnigg & Nielsen (2005)), our model does not treat the shares in the business as exogenous: on the contrary, these shares are determined within the model.

What are the other differences between our model and other models of contract in venture-backed projects? Firstly, our model implies that at the stage of fulfilling contract obligations the choice of effort from both parties is simultaneous. Due to the deficit of information and to the excessive potential costs of monitoring, neither VC nor entrepreneur can observe the level of effort of the counterparty in advance and then choose their own optimal levels of effort. Moreover, at the stage of fulfilling the contract both financing and financed parties are in somewhat equal conditions, since all important investments are already made. The entrepreneur has already made efforts to elaborate a business idea and the VC has financed this idea. Secondly, we include not only entrepreneurial effort, but also level of advice as a behavioural variable into our analysis. We thereby stress the importance of the VC's advice at the project realization stage. Schmidt (2003) uses a step-by-step model of equilibrium (the entrepreneur's level of effort is determined at an earlier stage, whereas the VC's optimal level of advice is revealed at the latter stage) and therefore concentrates on the entrepreneurial effort variable. We consider this approach as not fully appropriate because managerial advice is important in the life cycle of every project: the fact that such input adds value to every type of business is empirically proven.

Another particularity of our framework is that it is the venture capital fund that determines the shares in the business. There is nothing strange in this since it is logical that all the bargaining power must belong to the financing side. The specific investments which could result in the loss of the VC's bargaining power are made only after all the terms of the contract have been agreed. This means it is impossible for the entrepreneur to behave opportunistically (hence, he cannot acquire part of the VC's surplus). However, this may imply opportunistic behaviour on the financing side because the entrepreneur's specific investment (effort in elaborating the business idea) has been made before negotiations even start. Finally, the overwhelming bargaining power of the financing side can be explained by the fact that the financing side always has the choice of a wide number of projects, whereas entrepreneurs usually do not have any choice. According to statistics, out of 400 business ideas only one is financed.

Let us point out and explain the main drawbacks of our paradigm. Firstly, even if our approach to modelling the venture financing is correct, most of the parameters of our model are unobservable in real life. Probably the worst omission in our model is that it assumes completeness and symmetry of information. However, abstraction is a property of any model: therefore this type of omission is inevitable. In many models with uncertainty in the type of behavioural variables these variables are discrete (often such variables take two values: zero and some positive one). Continuity of effort variables adds value to our model and increases its explanatory power.

Another problem of our model is the complexity of the calculation due to a large number of parameters and equations (or inequalities). It is worth mentioning that the model is itself a simplified view of reality: in real life many more factors are considered. Moreover, our model has general (parametric) representation; hence, if we know all the parameters, the calculations will be minimal. In fact our model is not as complex as may seem at first sight: at some points it is actually oversimplified. The model does not consider such issues as implementing control rights (Hellmann (1998)) and the step-by-step format of negotiations (Lerner (1994)). We omitted such issues in our analysis because they do not affect the essence of venture financing and can potentially make our analysis too complex.

Despite the drawbacks of our model arising mostly from oversimplification, we managed to obtain some major results. The most important result is that we found the equilibrium levels of effort from the entrepreneur and the VC under the assumption of simultaneous choice of behavioural strategies. We showed that the obtained equilibrium is sustainable. This means that in the case of minor market failures (for example, due to incompleteness of information) and, consequently, agents' mistakes, the equilibrium will eventually return to the discovered point. Finding the equilibrium levels of effort was enough to establish an algorithm for finding the optimal shares in the business. These shares, being the most important part of the contract, can easily be found given the knowledge of all the parameters of the problem.

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APPENDIX

Finding the equilibrium (e(s),a(s)) from the reaction functions

From the reaction function of the VC (equation (5)) it follows that $e = (\frac{\gamma a^{\gamma-\beta}}{\beta s R})^{\frac{1}{\alpha}}$; $a = (\frac{\beta e^{\alpha} s R}{\gamma})^{\frac{1}{\gamma-\beta}}$

From the reaction function of the entrepreneur (equation (7)) it follows that $e = (\frac{\alpha a^{\beta}(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}}$; $a = (\frac{\delta e^{\delta-\alpha}}{\alpha(1-s)R})^{\frac{1}{\beta}}$

In order to find the optimal level of $a(s)$ equalize the values of e from the two reaction functions:

$$(\frac{\gamma a^{\gamma-\beta}}{\beta s R})^{\frac{1}{\alpha}} = (\frac{\alpha a^{\beta}(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}}$$

$$a^{\frac{\gamma-\beta}{\alpha}}[(\frac{\gamma}{\beta s R})^{\frac{1}{\alpha}} - (\frac{\alpha(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}} \cdot a^{(\frac{\beta}{\gamma-\alpha}-\frac{\gamma-\beta}{\alpha})}] = 0$$

$$a(s) = [\frac{(\frac{\gamma}{\beta s R})^{\frac{1}{\alpha}}}{(\frac{\alpha(1-s)R}{\delta})^{\frac{1}{\delta-\alpha}}}]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}} = [(\frac{\gamma}{\beta R})^{\frac{1}{\alpha}} \cdot (\frac{\alpha R}{\delta})^{\frac{1}{\alpha-\delta}}]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}} \cdot s^{\frac{\delta-a}{\alpha\gamma+\beta\delta-\delta\gamma}} \cdot (1-s)^{\frac{\alpha}{\delta\gamma-\alpha\gamma-\beta\delta}} = \theta s^{p}(1-s)^{q}$$

where $\theta = [(\frac{\gamma}{\beta R})^{\frac{1}{\alpha}} \cdot (\frac{\alpha R}{\delta})^{\frac{1}{\alpha-\delta}}]^{\frac{\alpha(\delta-\alpha)}{\alpha\gamma+\beta\delta-\delta\gamma}}, p = \frac{\alpha-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}, q = \frac{\alpha}{\delta\gamma-\alpha\gamma-\beta\delta}$

By analogy find the expression for $e(s)$:

$$(\frac{\beta e^{\alpha} sR}{\gamma})^{\frac{1}{\gamma-\beta}} = (\frac{\delta e^{\delta-\alpha}}{\alpha(1-s)R})^{\frac{1}{\beta}}$$

$$e^{\frac{\alpha}{\gamma-\beta}}[(\frac{\beta sR}{\gamma})^{\frac{1}{\gamma-\beta}} - (\frac{\delta}{\alpha(1-s)R})^{\frac{1}{\beta}} \cdot e^{(\frac{\delta-\alpha}{\beta}-\frac{\alpha}{\gamma-\beta})}] = 0$$

$$e(s) = [\frac{(\frac{\beta sR}{\gamma})^{\frac{1}{\gamma-\beta}}}{(\frac{\delta}{\alpha(1-s)R})^{\frac{1}{\beta}}}]^{\frac{\beta(\gamma-\beta)}{\delta\gamma-\alpha\gamma-\beta\delta}} = [(\frac{\gamma}{\beta R})^{\frac{1}{\beta-\gamma}} \cdot (\frac{\alpha R}{\delta})^{\frac{1}{\beta}}]^{\frac{\beta(\beta-\gamma)}{\alpha\gamma+\beta\delta-\beta\gamma}} \cdot s^{\frac{\beta}{\delta\gamma-\alpha\gamma-\beta\delta}} \cdot (1-s)^{\frac{\gamma-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}} = \xi s^{x}(1-s)^{y}$$

where: $\xi = [(\frac{\gamma}{\beta R})^{\frac{1}{\beta-\gamma}} \cdot (\frac{\alpha R}{\delta})^{\frac{1}{\beta}}]^{\frac{\beta(\beta-\gamma)}{\alpha\gamma+\beta\delta-\beta\gamma}}, x = \frac{\beta}{\delta\gamma-\alpha\gamma-\beta\delta}, y = \frac{\gamma-\beta}{\delta\gamma-\alpha\gamma-\beta\delta}$.

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