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Goran Andjelic,
Ivana Milosev,
*and Vladimir Djakovic**

EXTREME VALUE THEORY IN EMERGING MARKETS

ABSTRACT: *This paper investigates the performance of extreme value theory (EVT) with the daily stock index returns of four different emerging markets. The research covers the sample representing the Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes using the data from January 2006 – September 2009. In the paper a performance test was carried out for the success of application of the extreme value theory in estimating and forecasting of the tails of daily return distribution of the analyzed stock indexes. Therefore the main goal is to determine whether EVT adequately estimates and forecasts the tails (2.5% and 5% at the tail) of daily stock index return distribution in the emerging markets of Serbia, Croatia, Slovenia, and Hungary. The applied methodology during the research includes analysi s, synthesis and statistical/mathematical methods. Research results according to estimated Generalized Pareto Distribution (GPD) parameters indicate the necessity of applying market risk estimation methods, i.e. extreme value theory (EVT) in the framework of a broader analysis of investment processes in emerging markets.*

KEY WORDS: *Extreme Value Theory, Value at Risk, Risk Management, Generalized Pareto Distribution, Emerging Markets*

JEL CLASSIFICATION: D81, G10, G11

* Faculty of Technical Sciences, Department for Industrial Engineering and Management, Novi Sad, Serbia, *gandjelic@uns.ac.rs*, *ivanamilosev@yahoo.com*, *v_djakovic@uns.ac.rs*

1. INTRODUCTION

The modern terms of business activities in the financial markets condition the application of suitable methods of risk management. Events such as the financial crisis and financial market crash point to the inevitability of quantification and the estimation of the probability of occurrence of extremely high losses in investment activities. Currently Value at Risk (VaR) represents the most popular method of quantification and market risk management. Market risk is defined as the result of the price change of securities on the capital markets (Bessis, 2002).

The financial instability in the early 1970s generated the need for quantification of the market risks of the most important financial institutions. VaR was published by J P Morgan in 1994 as the method of risk management behind its Risk Metrics system. Theoretical ground for the VaR method was given by Jorion (1996), Duffie and Pan (1997) and Dowd (1998). It is defined as the worst loss over a target horizon with a given level of confidence (Jorion, 2001). VaR is a statistical measure of the maximal losses that can be incurred in investment activities, and losses that surpass the value of the VaR happen only with a certain probability (Linsmeier et al., 2000).

In spite of being established as an industry and regulatory standard i.e. in core financial areas such as portfolio optimization, capital allocation and risk limitation, VaR is often criticized for not being a coherent risk measure. In the VaR context, precise prediction of the probability of an extreme movement in the value of a portfolio is essential for both risk management and regulatory purposes (Gencay and Selcuk, 2004). With many different approaches and models the choice that VaR users face is the choice of picking the one that matches their purpose best. The approaches should make estimates that fit the future distribution of returns. If an overestimation of VaR is made, then operators end up with an overestimate of the risk. This could result in the holding of excessive amounts of cash to cover losses, as in the case with banks under the Basel II accord. The same is true in the opposite case, when VaR has been underestimated, resulting in failure to cover incurred losses. Statistical properties of the returns of assets such as volatility, kurtosis, and skewness are significant asset return characteristics.

The most common criticism, that the assumption that the profits/losses on a portfolio are normally distributed, is unrealistic. The theoretical ground that was provided by Mandelbrot[1,2] shows that financial return series exhibit leptokurtosis or 'heavier tails' than a normal distribution (Hauksson et al., 2000; Dacorogna et al., 2001). In essence, this means that any VaR calculation technique based on a normal distribution function will tend to give VaR estimates that are too low (Seymour and Polakow, 2003). Assuming normality when our data are heavy-tailed can lead to major errors in our estimates of VaR. Thus VaR will be underestimated at relatively high confidence levels and overestimated at relatively low confidence levels (Obadovic and Obadovic, 2009: 135).

Beyond the traditional approaches there is an alternative that uses the Extreme Value Theory (EVT) to characterize the tail behaviour of the distribution of returns. By focusing on extreme losses the EVT successfully avoids tying the analysis down to a single parametric family fitted to the whole distribution. Embrechts et al. (1997) and Mc-Neil and Fray (2000) survey the mathematical foundations of EVT and discuss its applications to financial risk management. The empirical results show that EVT-based models provide more accurate VaR estimates, especially in higher quantiles (Embrechts et al., 1999). For example, McNeil (1997), Harmantzis and Miao (2005) and Marinelli et al. (2007) show that EVT outperforms the estimates of VaR based on analytical and historical methods.

A special challenge is represented by the exploration of the possibilities of application, i.e. the performance of extreme value theory (EVT) on the financial markets of emerging countries, i.e. emerging markets. In the literature of the

1 Mandelbrot, B. (1963), 'New methods in statistical economics', *Journal of Political Economy*, Vol. 71, No. 5, pp. 421 – 440.

2 Mandelbrot, B. (1963), 'The variation of certain speculative prices', *Journal of Business*, Vol. 36, No. 4, pp. 394 – 419.

subject matter a fundamental difference exists between developed and emerging markets. Generally viewed, the world`s most developed stock exchange markets are considered more liquid and more efficient compared to those still emerging. In emerging markets such as Serbia there is also the case of a small number of data points (Drenovak and Urosevic, 2010).

The application of the EVT to emerging markets requires special attention, especially regarding insufficient liquidity, the small scale of trading, and, historically speaking, the asymmetrical and low number of trading days with certain securities. Financial theory indicates that higher volatility, which is characteristic for the returns of emerging markets, corresponds to higher expected returns on those markets (Salomons and Grootveld, 2003). Time series on financial markets often have the following characteristics: changing variability during time and empirical distribution that has tails that are heavier than tails of the normal distributions (Mladenovic and Mladenovic, 2006: 33). Also, compared to developed markets, emerging markets are characterized by capital market reforms, frequent internal and external financial shocks, a high level of country risk (i.e. political risk, economic risk, and financial risk), changes in credit rating, fluctuation of foreign exchange rates, a high level of insider trading, etc. Consequently economic activity in transition economies affected by the global crisis deteriorated much faster, from slowdown to rapid decline (Nuti, 2009). The previously listed factors considerably influence the increase of market volatility and consequently result in the increase of the divergence from normal distribution, which results in the impossibility to adequately predict the market risk, i.e. the emergence of extreme values in investment activities.

This paper tests the performance of application of the extreme value theory (EVT) on emerging markets of selected Central and Eastern European countries. Therefore the research goal is to determine whether EVT adequately estimates and forecasts the tails of daily return distribution in the emerging markets of Serbia, Croatia, Slovenia, and Hungary. Although different in many aspects, these countries have a common denominator being either EU member states or countries in the EU integration process. In this paper we examine the theoretical background and performance of EVT on Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes. EVT provides a formal framework for the study of the left and right tail behaviour of the fat-tailed return distributions. Namely, risk and reward are not equally likely to occur in these emerging markets.

The central objective of this paper is to test the performance of the application of the EVT on return series generated by the given stock indexes. Therefore the main motivation of this research is to provide up-to-date evidence on the risk management and return characteristics of emerging markets over time, i.e. to enable better forecast of out-of-sample events. Results of this research will be especially interesting to both domestic and foreign investors in global recessive business conditions. We present empirical evidence of the performance of application of the EVT in the emerging markets of the selected Central and Eastern European countries.

2. THEORETICAL BACKGROUND

The statistical analysis of extremes is essential for many of the risk management problems related to finance, i.e. investment processes. Extreme value theory (EVT) is the study of the tails of distributions and it is the key for sound risk management of financial exposures. Namely, forecast of the extreme movements that can be expected in financial markets, especially emerging ones, is tested within the framework of the EVT. The basic idea behind extreme value theory (EVT) is that in applications where only large movements are taken into consideration in some random variable, it may not be optimal to model the entire distribution of the event with all available data. Instead it may be better only to model the tails with tail events. Extreme value theory is a theory of the behaviour of large or extreme movements in a random variable, where extreme observations are used to model the tails of a random variable.

The family of extreme value distributions studies the limiting distributions of the sample extreme. This family can be presented under a single parameterization, known as the Generalized Extreme Value (GEV) distribution.

Definition 1. Let $\{X_i\}_{i=1}^{n}$ be a set of independent and identically distributed random variables with distribution function

$$F(x) := P\{X_i \le x\} \tag{1}$$

for any i. Also, we have to be able to assess the upper and lower tails of the distribution function F. Thus, consider the order statistics $M_n = \max\{X_1, X_2, ..., X_n\}$ and $m_n = \min\{X_1, X_2, ..., X_n\}$.

Both M_n and m_n are random variables that depend on the length n of the sample. Analogically with the Central Limit Theorem, we will be interested in the asymptotic behavior of these random variables as $n \to \infty$. Since $m_n = -\max\{-X_1, -X_2, ..., -X_n\}$, it is sufficient to state all the results for M_n, that is, focus on observations in the upper tail of the underlying distribution. The results for the lower tail will be straightforward to generalize.

The following theorem is a limit law first derived heuristically by Fisher and Tippett [3], and continued later by Gnedenko[4].

Theorem 1. Let $\{X_i\}_{i=1}^{n}$ be a set of n independent and identically distributed random variables with distribution function F and suppose that there are sequences of normalization constants, $\{a_n\}$ and $\{b_n\}$, such that, for some non-degenerated limit distribution F^*, we have

$$\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \le x\right) = \lim_{n\to\infty}\left[F(a_n x + b_n)\right]^n = F^*(x),\, x \in R \tag{2}$$

Then, there exist $\xi \in R$, $\mu \in R$ and $\sigma \in R_+$ such that $F^*(x) = \Gamma_{\xi,\mu,\sigma}(x)$ for any $x \in R$, where

[3] Fisher, R.A. and L.H.C. Tippett (1928), 'Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample', *Proceedings of the Cambridge Philosophical Society*, Vol. 24, pp. 180 – 190.

[4] Gnedenko, B.V. (1943), 'Sur la Distribution limite du Terme maximum d'une Série aléatoir', *Annals of Mathematics*, Vol. 44, No. 3, pp. 423 – 453.

$$\Gamma_{\xi,\mu,\sigma}(x) := \exp\left[-\left(1+\xi\frac{x-\mu}{\sigma}\right)_{+}^{-1/\xi}\right] \tag{3}$$

is the Generalized Extreme Value (GEV) distribution, which was first proposed in this form by von Mises[5]. The $1/\xi$ is referred to as the tail index, as it indicates how heavy the upper tail of the underlying distribution F is. When $\xi \to 0$, the tail index tends to infinity and $\Gamma_{\xi,\mu,\sigma}(x) \to \exp[-\exp(-(x-\mu)/\sigma)]$. Embrechts et al. (1997) describe GEV distribution in detail. Three fundamental types of extreme value distributions are defined by ξ:

1) If $\xi = 0$, the distribution is called the Gumbel distribution. In this case, the distribution spreads out along the entire real axis.
2) If $\xi > 0$, the distribution is called the Fréchet distribution. In this case, the distribution has a lower boundary.
3) If $\xi < 0$, the distribution is called the Weibull distribution. In this case, the distribution has an upper boundary.

The Fisher and Tippett theorem suggests that the asymptotic distribution of the maxima belongs to one of the three distributions above, regardless of the original distribution of the observed data. Random variables fall into one of three tail shapes, fat, normal, and thin, depending on the various properties of the distribution. Thus, the tails of distributions are:

- Thin. i.e. the tails are truncated.
- Normal. In this case, the tails have an exponential shape.
- Fat. The tails follow a power law.

It is a fact that financial returns are fat. The upper tail of any fat-tailed random variable (x) in EVT has the following property:

5 Von Mises, R. (1936), 'La Distribution de la plus grande de n Valeurs' in Selected Papers II. 1954. Providence, RI., *American Mathematical Society*, Vol. 2, pp. 271 – 294.

$$\lim_{t\to\infty}\frac{1-F(tx)}{1-F(t)}=x^{-\alpha}\text{, α>0, x>0} \tag{4}$$

where α is known as the tail index, and *F* (•) is the asymptotic distribution function. The reason why this is important is that, regardless of the underlying distribution of *x*, the tails have the same general shape, where only one parameter is relevant, i.e. α.

Theorem 2. Let $\{X_i\}_{i=1}^{n}$ be a set of n independent and identically distributed random variables with distribution function F. Define

$$F_u(y):=P(X\le u+y|X>u)=\frac{F(u+y)-F(u)}{1-F(u)}\text{, y>0} \tag{5}$$

as the distribution of excesses of X over the threshold u. Let x_F be the end of the upper tail of F, possibly a positive infinity. Then, if F is such that the limit given by Theorem 1 exists, there are constants $\xi\in R$ and $\beta\in R_+$ such that

$$\lim_{u\to x_F}\sup_{u<x<x_F}\left|F_u(x)-G_{\xi,\beta}(x-u)\right|=0\text{,} \tag{6}$$

where

$$G_{\xi,\beta}(y):=1-\left(1+\xi\frac{y}{\beta}\right)_+^{-1/\xi} \tag{7}$$

is known as the Generalized Pareto Distribution (GPD).

The application of EVT involves a number of challenges. First, the parameter estimates of the Generalized Extreme Value (GEV) distribution and GP limit distributions will depend on the number of extreme observations used. Second, the choice of a threshold should be large enough to satisfy the conditions that permit the application of Theorem 2, i.e. $u\to x_F$, while at the same time leaving a sufficient number of observations to render the estimation feasible (Bensalah, 2000).

Da Silva and de Melo Mendez (2003), Danielsson and de Vries (1997) and Embrechts et al. (1997) overviewed several empirical methods for estimation of tail thickness. The primary difficulty in estimating the tails is the determination of the start of the tails. Characteristically, these estimators use the highest/lowest realizations to estimate the parameter of tail thickness, which is called the tail index. The moments based estimator for the tail index was proposed by Hill.[6] The estimator is conditional on knowing how many extreme order statistics for a given sample size have to be taken into account. The tail index is estimated by using the most extreme observations above a threshold S_n, where n is the sample size. The most common estimator of the tail index is the Hill estimator, which is generally considered to have more desirable properties than other estimators. The efficient determination of the tail threshold, S_n, requires an optimal assessment of the trade–off between bias and variance (Danielsson and de Vries, 2002).

In our study the performance of EVT is analyzed in emerging markets of the selected Central and Eastern European countries. Zikovic and Aktan (2009) investigated the relative performance of a wide array of VaR models with the daily returns of the Turkish and Croatian stock index. They concluded that only advanced and theoretically sound VaR models such as EVT and HHS can adequately measure equity risk on the Turkish and Croatian equity markets in times of crisis. Similarly Gencay and Selcuk (2004) examined the relative performance of VaR models with the daily stock market returns of nine different emerging markets. Coronel-Brizio and Hernandez-Montoya (2005) investigated the so-called Pareto-Levy or power-law distribution as a model to describe probabilities associated with extreme variations of worldwide stock market indexes data. Embrechts et al. (1999) examined the role of extreme value theory as an important methodological tool for securitization of risk and alternative risk transfer. Da Silva and de Melo Mendez (2003) used the extreme value theory to analyze ten Asian stock markets, identifying which type of extreme value asymptotic distribution better fits historically extreme market events. They concluded that the extreme value method of estimating VaR is a

6 Hill, B.M. (1975), 'A simple general approach to inference about the tail of a distribution', *The Annals of Statistics*, Vol. 3, No. 5, pp. 1163 – 1174.

more conservative approach to determining capital requirements than traditional methods. Mladenovic and Mladenovic (2006) investigated and exhibited the evaluation of value parameters regarding risk based on analysis of the specific financial time series. They investigated the daily return data of share prices of CISCO and INTEL companies as well the NASDAQ market index, and concluded that one of the key elements in application of the extreme value theory is determining a threshold value, and consequently a group of extreme values. Drenovak and Urosevic (2010) investigated the Serbian market using the Svensson parametric model, taking into account issues specific to emerging markets in general and the Serbian market in particular. They argue that no risk management or asset/liability model, the cornerstones of the contemporary financial industry, can be implemented without regular use of benchmark spot curve estimates.

The contribution of this paper is the empirical investigation and analysis of extreme value theory (EVT) on the daily stock index returns of four different emerging markets, while estimating and forecasting the tails of the daily return distribution of the tested stock indexes.

3. METHODOLOGY REVIEW

This part of the paper presents the research methodology that is particularly focused on the performance analysis of the application of the extreme value theory (EVT) on the emerging markets of Serbia, Croatia, Slovenia and Hungary, i.e. in investment activities. Volatile markets provide an appropriate environment to study the performance of the EVT. The high volatility and thick-tail nature of the Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes provide an adequate platform to test the performance of application of the extreme value theory (EVT) in the emerging markets of selected Central and Eastern European countries.

In the paper a performance test was carried out, i.e. the success of the usage of the extreme value theory (EVT) in the estimation and forecasting of the tails of daily return distribution of the analyzed stock indexes. The movements of the returns of the observed stock indexes in emerging markets were analyzed, i.e.

the losses (the left tail) and the profits (the right tail) of investment activities. EVT was used to forecast the largest expected decrease and increase in each data series over a given period. In this way it is possible to test the performance of the application of the EVT as a market risk quantifier on the one hand and as determining the reward on the other. The goal of the research is to consider the performance of the application of the extreme value theory (EVT) for estimating one period ahead return prediction in both tails (2.5% and 5% at the tail) of the return distribution in emerging markets. The sample of the research comprises daily returns of stock indexes of selected Central and Eastern European countries, i.e. Serbia, Croatia, Slovenia, and Hungary. The tested stock indexes are respectively BELEXline, CROBEX, SBI20, and BUX, during the period 10.01.2006 – 31.08.2009. The research was conducted during this period due to the accessibility of historical data for the stock indexes of the selected emerging markets, and to performed improvements in the development of the BELEX trading system in 2005. In addition this time frame comprises the data prior and during the period of the world economic crisis, i.e. recessive business conditions, which contributes to adequate analysis of the performance of application of the EVT. The applied methodology used during the research includes analysis, synthesis and statistical/mathematical methods. In the analysis the Generalized Pareto Distribution (GPD) was used, which provided a basis for observing two approaches in estimating risk and reward, i.e. the emergence of extreme return values in investment activities. Returns are divided into two groups, the first group comprised of returns having values less than zero (the left tail), and the second group comprised of returns having values more than zero (the right tail). Therefore the analysis is performed for each tail of the fat-tailed return distribution separately. With the analysis of the left tail, the possibility of estimating the maximal loss was tested, while with the analysis of the right tail, the possibility of estimating the maximal profit in the investment activity was tested. From a risk management point of view, the estimated return at the left tail determines the amount of capital that should be allocated to cover the possible loss. Also the estimated return at the right tail is significant for investors, especially regarding the profitability of investment activities.

The returns on the stock indexes tested in this paper are calculated as

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} \tag{8}$$

where

R_t – return on stock index during a period t,
P_t – stock index price during a period t,
P_{t-1} – stock index price during a period t-1.

The changes in the daily returns of the stock indexes point to the specificities, i.e. characteristics of the observed emerging markets with a special accent on investment possibilities and market risks as the determinants of such activities. With the analysis of the success of the Generalized Pareto Distribution (GPD), the behaviour of the emerging markets is tested in terms of volatility and probability of extreme value occurrences. The in-sample period comprises the period between 10.01.2006 and 31.08.2007, while the out-of-sample period comprises the period between 01.01.2008 and 31.08.2009. On the basis of the in-sample period the threshold value was calculated, according to which the returns value of the following day was tested. The returns value was successfully estimated in case the returns value of the following day was higher than the estimated value for the left tail and less than the estimated value for the right tail. For the opposite the estimation was unsuccessful.

Our forecasting methodology is such that we analyze the application of two approaches. The first approach understands a sliding window of two years' daily returns data (limited interval). In the calculation of the GPD, as the daily returns of the following day were added, the oldest daily returns were cast out from the observed window. This sliding window has 415 days as a basis for GPD calculation and it is divided into left and right tails. For example, with a window size of 415, the window is placed between the 1st and the 415th data points, the model is estimated, and the return forecast is obtained for the 416th day at different quantiles. Next, the window is moved one day ahead to the 2nd and

416^{th} data points to obtain a forecast of the 417^{th} day return with updated parameters from this new sample.

The second approach understands an interval of 415 days, in such a way that the interval is increased after testing each of the following days, and which is added to the in-sample without casting out the oldest daily returns (long interval). In this way the number of the days is increased, according to which the model estimation and the daily return forecast is carried out. That is, this approach does not utilize a window and uses all available data starting at the 415^{th} day. For instance, the model is estimated adding the 416^{th} day return into the sample and a forecast of the 417^{th} day return is obtained and stored. Since it is practically impossible to determine an optimum parameterization or a threshold value for each approach (optimal threshold determination), i.e. instead of determining a threshold value at each step, we utilized 2.5% and 5% at the tail of the observed sample in both GPD approaches. At the beginning of the analysis the distribution of the sample has been tested with the Kolmogorov-Smirnov test, with the objective of determining whether the sample has normal distribution. On the basis of the central dispersive parameters, the picture of the distribution of the sample was gained. The normal distribution of the sample means that the coincidental variable (x), with the arithmetical middle μ and the standard deviation σ, is normally distributed in case the function of probability f (x) gives the variable (x) the value of X, following the next function of probability:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{(x-\mu)}{\sigma} \right)^2 \right] \tag{9}$$

where

σ - standard deviation,
π – pi, constant = 3,14159…,
μ – arithmetical middle.

Also, during the testing of the sample, its characteristics have been examined - skewness and kurtosis. Their coefficients have been calculated according to the next formulae:

$$\text{Coefficient skewness} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}_n\right)^3}{\left(\overline{S}_n\right)^3} \tag{10}$$

$$\text{Coefficient kurtosis} = \frac{\frac{1}{n}\sum_{k=1}^{n}\left(X_i - \overline{X}_n\right)^4}{\left(\overline{S}_n^{\,2}\right)^2} \tag{11}$$

where

X_i - sample,
$\overline{X}_n$ - middle of the sample,
$\overline{S}_n$ - dispersion of the sample.

The Kolmogorov–Smirnov test is used to test whether two underlying one-dimensional probability distributions differ. The random process F(x) is formed as the estimation problem and q used as the test statistic:

$$q = \max_x |\hat{F}(x) - F_0(x)| \tag{12}$$

This choice is based on the following observations: For a specific ζ, the function $\hat{F}$ (x) is the empirical estimate of (x). It tends, therefore, to F(x) as $n \to \infty$. From this it follows that:

$$E(\hat{F}(x))=F(x) \qquad \hat{F}(x) \to F(x), (n \to \infty) \tag{13}$$

This shows that for large n, q is close to 0 if H_0 is true and it is close to F(x) – $F_0(x)$ if H_1 is true. It leads, therefore, to the conclusion that we must reject H_0 if q is larger than some constant c. This constant is determined in terms of the significance level $\alpha = F\{q > c|H_0\}$ and the distribution of q. Under hypothesis H_0, the test statistic q is used. Using the Kolmogorov approximation, we obtain:

$$\alpha = P\{q > c|H_0\} = 1 - e^{-2nc^2} \tag{14}$$

The test thus proceeds as follows: Form the empirical estimate $\hat{F}$ (x) of F(x);

$$\text{Accept } H_0 \text{ if } q > \sqrt{-\frac{1}{2n}\ln\frac{\alpha}{2}} \tag{15}$$

The resulting Type II error probability is reasonably small only if n is large.

The research carried out in the paper understands the analysis of the performances of the named calculation approaches of the GPD in the selected emerging markets, i.e. the adequacy of the usage of the limited or the long interval. Due to the previously mentioned characteristics of emerging markets in the introduction to the paper, it was not possible to apply a standard approach to this problem, when for the in-sample period a fixed period of a different number of days was taken. Namely, there is simply not enough data to do sensible analysis in the selected emerging markets, whether by EVT or any other method. The global economic crisis represents a special problem, which especially started to manifest itself in the observed emerging markets from September 2008, which additionally influenced the shortening of the period in which the results of the application of the EVT could be analyzed.

4. DATA AND PRELIMINARY ANALYSIS

Due to data availability and the possibility of its dynamic processing and monitoring, i.e. the performance analysis of application of the extreme value theory (EVT) in the emerging markets of Serbia, Croatia, Slovenia, and Hungary, the specimen in the research comprises daily stock index returns from these markets for the period 10.01.2006 – 31.08.2009 (934 days). In order to investigate the risk and reward dynamics in selected emerging markets the data set are the daily closings of the Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes in the observed period.

Table 1 provides descriptive statistics of daily returns, computed as $r_t = \ln(1+R_t) = \ln\frac{P_t}{P_{t-1}}$. Daily sampling is chosen in order to capture high-frequency fluctuations in return processes that may be critical for identification of rare events in the tails of distribution, while avoiding modeling the intraday return dynamics, abundant with spurious emerging market microstructure distortions and trading frictions.

Table 1. Descriptive statistics of the daily returns in the period 10.01.2006 – 31.08.2010 (934 days)

Stock index	Range	Min	Max	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Stat.	Stat.	Stat.	Stat.	Std. Error	Stat.	Stat.	Stat.	Std. Error	Stat.	Std. Error
BELEXline	16.84	-6.97	9.87	-0.0389	0.04026	1.23038	1.514	0.368	0.08	10.076	0.16
CROBEX	25.54	-10.76	14.78	-0.0002	0.05926	1.81105	3.28	-0.113	0.08	8.981	0.16
SBI20	15.98	-8.3	7.68	-0.0105	0.04476	1.36794	1.871	-0.681	0.08	7.339	0.16
BUX	25.83	-12.65	13.18	-0.0158	0.06658	2.03477	4.14	-0.089	0.08	6.22	0.16

Source: Authors' calculations

For emerging countries a significant problem for a serious and statistically significant analysis is the short history of their market economies and active trading in financial markets. Due to the short time series of the returns of some stocks of the selected emerging markets, the research in the paper comprises detailed analysis of the stock indexes of the observed countries. The stock indexes can be observed as a portfolio of the selected stocks of each emerging market. Thus data used in the performance analysis of the application of the extreme value theory (EVT) are the daily return series from the Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes.

The data are collected from each official stock exchange web site. At the beginning of the research a test of normal distribution was carried out, where it was tested whether the returns of the stock indexes (data) have normal distribution. According to the Kolmogorov-Smirnov test, it can be said with great certainty that stock indexes are not normally distributed, i.e. there are considerable differences between the in-sample distribution and normal distribution. On the basis of the central dispersive parameters an image of the sample distribution was achieved. The Kolmogorov-Smirnov test shows that none of the observed stock indexes have normal distribution. Also the values of skewness and kurtosis in Table 1 indicate that returns deviate from normality. Table 2 shows the results of normal distribution. On the basis of the parameters of descriptive statistics, the biggest difference in the daily returns (max-min) can be seen at BUX and CROBEX, while the difference is less at BELEXline and SBI20.

Table 2. Kolmogorov – Smirnov tests of Normality for the stock indexes in the period 10.01.2006 – 31.08.2010

		BELEXline	CROBEX	SBI20	BUX
N		934	934	934	934
Normal Parameters	Mean	-3.8908	-2.2484	-1.0514	-1.5782
Std. Deviation		1.2304	1.8111	1.3679	2.0348
Most Extreme Differences	Absolute	0.114	0.12	0.108	0.075
	Positive	0.114	0.097	0.088	0.075
	Negative	-0.11	-0.12	-0.108	-0.063
Kolmogorov-Smirnov Z		3.472	3.653	3.315	2.306
Asymp. Sig. (2-tailed)		.000	.000	.000	.000

a - Test distribution is Normal.
b - Calculated from data.
Source: Authors' calculations

The parameters of Standard Deviation, Variance, Skewness, and Kurtosis point to the basic characteristics of the sample (Table 1). According to Skewness, we perceive that the curve of the stock index of the SBI20 has an asymmetric image and the distribution curve bends towards the higher values (to the right). The results are identical for CROBEX and BUX, while the values in the BELEXline are such that the curve has an asymmetric image and bends toward the lower values (to the left). The listed parameters show that the value changes in the stock indexes of the SBI20, CROBEX, and BUX in most of the cases (days) are positive, while at BELEXline there is a higher number of negative changes. According to Standard Deviation, we can observe that the returns at BELEXline and SBI20 are more homogenous, because the Standard Deviation values are less, while the returns are less homogenous at BUX and CROBEX, which is mirrored in the higher values of Standard Deviation. The values of Minimum and Maximum show the deviations of the minimal and maximal returns.

We also analyzed the QQ-plots of returns against the exponential distribution for each stock index. These plots confirm that the return distributions have fat tails. In statistics, a quantile-quantile plot (QQ plot) is a convenient visual tool for examining whether a sample comes from a specific distribution. Namely, the quantiles of an empirical distribution are plotted against the quantiles of a hypothesized distribution. If the sample comes from the hypothesized distribution or a linear transformation of the hypothesized distribution, the QQ plot is linear. In the extreme value theory (EVT) and applications, the QQ plot is typically plotted against the exponential distribution to measure the fat-tailedness of a distribution. If the data are from an exponential distribution, the points on the graph will lie along a straight line. If there is a concave presence, this indicates a fat-tailed distribution, whereas a convex departure is an indication of short-tailed distribution.

Table 3. Daily returns characteristics (left tail) of the stock indexes in the period 10.01.2006 – 31.08.2010

Stock index	Less than -8%	% of the total sample	-8% to -6%	% of the total sample	-6% to -3%	% of the total sample	-3% to -2%	% of the total sample	-2% to -1%	% of the total sample	-1% to 0%	% of the total sample
BELEXline	0	0	2	0.21	14	1.5	20	2.14	100	10.71	362	38.76
CROBEX	2	0.21	6	0.64	36	3.85	44	4.71	80	8.57	283	30.3
SBI20	2	0.21	3	0.32	18	1.93	30	3.21	86	9.21	331	35.44
BUX	4	0.43	5	0.54	41	4.39	56	6	140	14.99	231	24.73

Source: Authors' calculations

According to the distribution of the stock index returns (Tables 3, 4 and 5), we perceive there are a small number of daily returns with extremely low values in BELEXline, while BUX has the most negative daily returns in the interval, from -8% to -1%. BELEXline has the most days with negative returns, followed by BUX and SBI20, and CROBEX has the least number of negative returns.

BELEXline has a high number of days with negative returns, and most of the returns are in the intervals from -1% to 0% and from -2% to -1%.

Table 4. Daily returns characteristics (right tail) of the stock indexes in the period 10.01.2006 – 31.08.2010

Stock index	0% to 1%	% of the total sample	2% to 3%	% of the total sample	3% to 6%	% of the total sample	6% to 8%	% of the total sample	More than 8%
BELEXline	327	35.01	76	8.14	22	2.36	9	0.96	2
CROBEX	294	31.48	113	12.1	46	4.93	26	2.78	4
SBI20	328	35.12	95	10.17	29	3.1	10	1.07	2
BUX	240	25.7	114	12.21	54	5.78	45	4.82	4

Source: Authors' calculations

Compared to BELEXline, SBI20 has many less days with negative returns in the intervals from -1% to 0% and -2% to -1%. CROBEX has the greatest number of positive returns of all observed stock indexes. A large number of these returns belong to the intervals of less than -8%, from -8% to -6% and from -3% to -2%. BUX is characterized by a high number of negative returns when compared to other stocks indexed, and is so in intervals of less than -8%, from -8% to -6%, from -6% to -3%, and from -3% to -2%.

Table 5. Daily returns characteristics (left and right tail-summary) of the stock indexes in the period 10.01.2006 – 31.08.2010

Stock index	-8% to 0%	% of the total sample	0% to 8%	% of the total sample
BELEXline	498	53.32	436	46.68
CROBEX	451	48.29	483	51.71
SBI20	470	50.32	464	49.68
BUX	477	51.07	457	48.93

Source: Authors' calculations

By analyzing the tables and figures (Appendices), a trend of value changes can be observed for all of the stock indexes, i.e. tail estimation by years. During the tail estimation the returns of the last two years were observed (out-of-sample, 01.01.2008 – 31.08.2009) and the threshold value was obtained on the basis of the Generalized Pareto Distribution (GPD). In Table A1-1 and Table A1-2 the threshold values are shown for the in-sample period of 2006-2007. The analysis of the left tail shows that the threshold value was the least at BELEXline, SBI20, CROBEX, and BUX, respectively. The analysis of the right tail indicates that the threshold values were very alike for BELEXline and SBI20, while the same was higher at CROBEX and BUX, respectively. The reasons for these threshold value distributions lie in the distribution of daily returns and their values. Namely, BELEXline has a great number of negative returns but with less value, while SBI20 has a lower number of negative returns but with higher values (Tables 3, 4 and 5). Also, threshold values direct attention to the aforementioned characteristics of the observed stock indexes.

The threshold value calculated on the basis of a higher number of days (long interval, Tables A3-1 and A3-2) was less when compared to the threshold value calculated on the basis of a fewer number of days (limited interval, Tables A2-1 and A2-2) in all observed stock indexes. Less threshold value is obtained in the case of the GPD calculation approach with the long interval being applied. Namely, it has been proven that the approach of the GPD calculation is more rigorous when the approximation is carried out on returns with a higher number of days (long interval), which causes a lower threshold value as a consequence of dilution of extreme return values (Tables A3-1 and A3-2). By analyzing the performances of the application of the long interval in calculating the GPD, it is concluded that the individual extreme values of returns have less effect on the threshold value than when applying the approach that understands the application of the limited interval (Tables A4-1, A4-2, A4-3 and A4-4). Namely, the extreme values of returns move the threshold value to the left side for the negative values, i.e. the left tail, while a movement is seen towards the right side, i.e. the right tail when the approach of GPD calculation is applied which understands the application of the limited interval. Bearing in mind the formerly stated, the success of the application of the GPD calculation approach, which understands the application of the long interval in estimating the extreme values, is undoubtedly significant, with the limiting effect that on the majority of days the threshold value is significantly higher from the daily values of the returns. At the left and right tails, BELEXline has the least threshold values, followed by SBI20, CROBEX, and BUX, respectively. For the left tail in 2006 and in the period 2006-2007 it was characteristic of the threshold value of CROBEX to be nearer to the threshold value of BELEXline and SBI20, while in the periods 2006-2008 and 2006-2009 the same was closer to the threshold values of the BUX. For the right tail, all threshold values are characterized by the same trend, with the exception that the threshold values of BUX are higher than in the other stock indexes. In the period 2006-2009 the threshold values did not increase at the left and the right tail, but kept their values from the previous period.

5. RESULTS AND DISCUSSION

In this section of the paper the results of the research based on GPD estimates for the BELEXline, CROBEX, SBI20, and BUX stock indexes are to be presented and analyzed. The analysis is performed for each distribution tail (2.5% and 5% at the tail) separately, to test the estimation possibilities of the maximal loss (left tail) and maximal profit (right tail) in investment activities. The research includes the performance analysis and application adequacy of two calculation approaches of GPD, i.e. the limited (two years' daily returns data) and the long interval on the selected emerging markets. The returns value in investment activities is successfully estimated in the case where the returns value of the following day is higher than the left tail estimate, but less than the right tail estimate. Otherwise the estimation was unsuccessful.

Table 6. Performance testing of the GPD application for BELEXline in the period 2008-2009 - limited interval

BELEXline	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	249	93.96	244	91.73	142	94.04	135	90.00
Unsuccessful	16	6.04	22	8.27	9	5.96	15	10.00
Total	265	63.70	266	63.94	151	36.30	150	36.06

Source: Authors' calculations

In Tables 6 and 7, the results of the aforementioned approaches of GPD calculation are presented for BELEXline in the period 2008-2009. At BELEXline there are no significant differences in the percentage of success of the estimations in predicting the values of the returns by the approaches calculating the GPD, i.e. the difference is not higher than 2%. The difference in percentage is higher at the right tail (5% at the tail) and it is 5.33%. The results show that the estimate is more successful with the limited than with the long interval calculation of the GPD.

Table 7. Performance testing of GPD application for BELEXline, 2008-2009 -long interval

BELEXline	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	246	92.83	239	89.85	140	92.72	127	84.67
Unsuccessful	19	7.17	27	10.15	11	7.28	23	15.33
Total	265	63.70	266	63.94	151	36.30	150	36.06

Source: Authors' calculations

In Tables 8 and 9, the results of the application of the mentioned approaches of GPD calculation are presented for CROBEX in the period 2008-2009.

Table 8. Performance testing of the GPD application for CROBEX in the period 2008-2009 - limited interval

CROBEX	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	197	91.20	180	83.33	184	93.40	173	87.82
Unsuccessful	19	8.80	36	16.67	13	6.60	24	12.18
Total	216	52.30	216	52.30	197	47.70	197	47.70

Source: Authors' calculations

At CROBEX there is a difference of some 4% in the results of the success of the returns value estimation of the approach of calculating the GPD, both for the left and the right tails. The number of unsuccessful estimations was less using the limited interval than using the long interval of calculating the GPD.

Table 9. Performance testing of the GPD application for CROBEX in the period 2008-2009 - long interval

CROBEX	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	190	87.96	171	79.17	180	91.37	165	83.76
Unsuccessful	26	12.04	45	20.83	17	8.63	32	16.24
Total	216	52.30	216	52.30	197	47.70	197	47.70

Source: Authors' calculations

In Tables 10 and 11, the results of the application of the mentioned approaches of GPD calculation are presented for SBI20 in the period 2008-2009.

Table 10. Performance testing of the GPD application for SBI20 in the period 2008-2009 - limited interval

SBI20	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	210	91.70	202	87.83	166	91.21	163	89.56
Unsuccessful	19	8.30	28	12.17	16	8.79	19	10.44
Total	229	55.58	230	55.83	182	44.17	182	44.17

Source: Authors' calculations

The results of the success of the value estimations of the returns in both approaches of calculating the GPD are almost identical at SBI20. The differences are minimal both in the case of the left and the right tail, because the percentage of unsuccessful estimations are around and less than 1.5%.

Table 11. Performance testing of the GPD application for SBI20 in the period 2008-2009 - long interval

SBI20	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	209	91.27	198	86.09	167	91.76	163	89.56
Unsuccessful	20	8.73	32	13.91	15	8.24	19	10.44
Total	229	55.58	230	55.83	182	44.17	182	44.17

Source: Authors' calculations

In Tables 12 and 13, the results of the application of the mentioned approaches of GPD calculation are presented for BUX in the period 2008-2009.

Table 12. Performance testing of the GPD application for SBI20 in the period 2008-2009 - limited interval

BUX	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	199	92.13	191	88.43	179	89.95	167	83.92
Unsuccessful	17	7.87	25	11.57	20	10.05	32	16.08
Total	216	52.05	216	52.05	199	47.95	199	47.95

Source: Authors' calculations

The results of the success of the value estimations of the returns in both approaches of calculating the GPD are almost identical at BUX. The differences are minimal both in the case of the left and the right tail, because the percentage of unsuccessful estimations are around and less than 2%.

Table 13. Performance testing of the GPD application for SBI20 in the period 2008-2009 - long interval

BUX	Left tail				Right tail			
2008-2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	201	93.06	192	88.89	176	88.44	167	83.92
Unsuccessful	15	6.94	24	11.11	23	11.56	32	16.08
Total	216	52.05	216	52.05	199	47.95	199	47.95

Source: Authors' calculations

The results of the research in the period 2008-2009 (Tables 6, 7, 8, 9, 10, 11, 12 and 13) for the left tail (2.5% at the tail) show that the least number of unsuccessful estimations are to be found at BELEXline, 6.04% (limited interval) and 7.17% (long interval); followed by the results at BUX, 7.87% (limited interval) and 6.94% (long interval); SBI20, 8.30% (limited interval) and 8.73 % (long interval); and finally at CROBEX, 8.8% (limited interval) and 12.04% (long interval). For the left tail (5% at the tail) the least number of unsuccessful estimation results are to be found at BELEXline, 8.27% (limited interval) and

10.15% (long interval), as well as at BUX, 11.57% (limited interval) and 11.11% (long interval). The next in line is SBI20 with 12.17% (limited interval) and 13.91% (long interval); and finally CROBEX with 16.67% (limited interval) and 20.83% (long interval) of unsuccessful estimations.

The results of the research in the period 2008-2009 (Tables 6, 7, 8, 9, 10, 11, 12 and 13) for the right tail (2.5% at the tail) show that the least number of unsuccessful estimations are to be found at BELEXline, 5.96% (limited interval) and 7.28% (long interval); followed by the results at CROBEX, 6.60% (limited interval) and 8.63% (long interval). They are followed by SBI20, 8.79% (limited interval) and 8.24 % (long interval), and finally at BUX, 10.05% (limited interval) and 11.56% (long period) of unsuccessful estimations. For the right tail (5% at the tail) the least number of unsuccessful estimation results are to be found at SBI20, 10.44% (limited interval) and 10.44% (long interval); followed by BELEXline, 10.00% (limited interval) and 15.33% (long interval); CROBEX with 12.18% (limited interval) and 16.24% (long interval); and finally, BUX with 16.08% (limited interval) and 16.08% (long interval) of unsuccessful estimations.

The results of the research for 2008 (Tables A5, A6, A9, A10, A13, A14, A17 and A18) for the left tail (2.5% at the tail) show that the least number of unsuccessful estimations are to be found at BELEXline, 9.25% (limited interval) and 10.40% (long interval); followed by the results at BUX, 11.35% (limited interval) and 9.22% (long interval). They are followed by SBI20, 12.84% (limited interval) and 13.51 % (long interval), and finally at CROBEX 13.04% (limited interval) and 18.12% (long period). For the left tail (5% at the tail) the least number of unsuccessful estimation results are to be found at BELEXline, 11.05% (limited interval) and 13.37% (long interval); followed by BUX, 17.02% (limited interval) and 14.89% (long interval); SBI20 with 18.79% (limited interval) and 20.81% (long interval); and finally CROBEX, with 24.64% (limited interval) and 28.26% (long interval) of unsuccessful estimations.

The results of the research for 2008 (Tables A5, A6, A9, A10, A13, A14, A17 and A18) for the right tail (2.5% at the tail) show that the least number of unsuccessful estimations are to be found at CROBEX, 8.11% (limited interval) and 8.11% (long interval), followed by the results at BELEXline, 8.97% (limited interval) and 8.97% (long interval). The next in line is BUX, 12.84% (limited interval) and 12.84 % (long interval), and finally at SBI20, 14.43% (limited interval) and 13.40% (long period) of unsuccessful estimations. For the right tail (5% at the tail) the least number of unsuccessful estimation results are to be found at BELEXline, 10.26% (limited interval) and 12.66% (long interval); followed by CROBEX, 12.61% (limited interval) and 14.41% (long interval); SBI20 with 15.46% (limited interval) and 15.46% (long interval); and finally BUX, with 15.60% (limited interval) and 15.60% (long interval) of unsuccessful estimations.

For all tested stock indexes in 2008, the number of days with negative returns (left tail) is considerably higher than those with positive returns (right tail). In 2008, at BELEXline the results show that the differences in successful estimations of the returns value approaches of GPD calculations are within 3% at the left tail, while the difference at the right tail is 2.5 %, and as such it can be concluded that the success is almost the same (Tables A5 and A6). At CROBEX the analysis of the results for both approaches of GDP calculation show that the differences of the successful estimation of the returns value are within 5.08% at the left tail, while the difference at the right tail is within 2%, and as such it can be concluded that the success of both approaches is almost the same (Tables A9 and A10). The analysis of the results of the differences for both approaches of GDP calculation of the successful estimation of the returns value at SBI20, show that there are no considerable differences in successfully estimates, i.e. there are no significant differences in the application of the two approaches to the investment process (Tables A13 and A14). In Tables A17 and A18, the results of the differences of the successful estimation of the returns value approaches of GPD calculation are shown at BUX in 2008. The results show that at the right tail the difference is within 2.13% (2.5% at the tail), while there are no differences of the same at 5% at the tail. In addition the results show that the estimations are more successful in the case of the long interval.

The research results in 2009 (Tables A7, A8, A11, A12, A15, A16, A19 and A20) for the left tail (2.5% at the tail) show that at SBI20 there are no unsuccessful estimations using either of the two approaches, followed by BELEXline with 0%

of unsuccessful estimations (limited interval) and 1.09% (long interval). The next is CROBEX with 1.28% (limited interval) and 1.28% (long interval); and finally, BUX with 1.33% (limited interval) and 1.33% (long interval) of unsuccessful estimations. For the left tail (5% at the tail) the least percentage of unsuccessful estimations is at SBI20, 0% (limited interval) and 1.23% (long interval); then BELEXline, 1.09% (limited interval) and 2.17% (long interval); BUX, 2.67% (limited interval) and 2.67% (long interval); and finally CROBEX with 2.56% (limited interval) and 7.69% (long interval) of unsuccessful estimations.

The research results in 2009 (Tables A7, A8, A11, A12, A15, A16, A19 and A20), for the right tail (2.5% at the tail) show that the least number of unsuccessful estimations is at SBI20, 2.35% (limited interval) and 2.35% (long interval), followed by BELEXline with 2.74% unsuccessful estimations (limited interval) and 5.48% (long interval). The next is CROBEX with 4.65% (limited interval) and 9.30 (long interval), and finally BUX with 6.67% (limited interval) and 10.00% (long interval) of unsuccessful estimations. For the right tail (5% at the tail) the least percentage of unsuccessful estimations is at SBI20, 4.71% (limited interval) and 4.71% (long interval); then BELEXline, 9.72% (limited interval) and 20.55% (long interval); CROBEX, 11.63% (limited interval) and 18.60% (long interval); and finally BUX with 16.67% (limited interval) and 16.67% (long interval) of unsuccessful estimations.

In 2009 the difference in the success of GPD application, i.e. in estimating the value of returns, is minimal at the left tail while at the right tail (2.5% at the tail) it is 2.74%, and at the same (right) tail (5% at the tail) the difference is 10.83% (Tables A7 and A8). Also at the right tail the number of unsuccessful estimations is less when the limited interval approach is used, while the number of unsuccessful estimations is higher using the long interval approach in calculating the GPD. At CROBEX in 2009 there are no differences in the success of estimating the value of the returns in GPD calculations at the left tail (2.5% at the tail) and for 5% at the tail it is 5.13%. For the right tail there is a difference within 7% (Tables A11 and A12). At the right tail there are less unsuccessful estimations using the limited interval, while the number is higher using the long interval for calculating the GPD. In 2009, by result analysis from Tables A15 and A16, it can be concluded that at SBI20 the difference in the successful estimations of the returns value in GPD calculations in the investment process is 1.23%. In Tables A19 and A20 the results of the success of the GPD calculation approaches are shown for BUX for the year 2009. The results show that there are no differences in the success of the value estimations of the returns in the GPD calculations for the left tail, while for the right tail it is 3.33% (2.5% at the tail). At the right tail the number of unsuccessful estimations is less using the limited interval and is higher when the long interval of GPD calculations are used. The results show that at BELEXline, CROBEX, SBI20, and BUX the success of estimating the returns was higher at the threshold value that was calculated at the limited interval. This fact is the consequence of the volatility peculiarities of emerging markets. The volatility of these markets point to the necessity of the adequate specification of the limits in threshold calculations (optimal threshold determination), and this is especially true if the estimation successes of the returns values of the named approaches of GPD calculations are taken into consideration. The threshold value in highly volatile circumstances has to be adequate, i.e. it cannot be too high in relation to the days with less volatility (slight changes in daily returns), because this has a direct impact on the performance of the tested approaches.

On the basis of result comparisons from 2008 and 2009 it can be concluded that the number of successful estimations is considerably higher in 2009. The listed results are the consequences of the decreased trade turnover in the selected emerging markets, as well of a more restrictive approach toward investment activities under the conditions of the global economic crisis. Namely, as a consequence of the crisis, the trade turnover on the stock markets of the selected emerging markets decreased abruptly, resulting in a high success of GPD applications in estimating the returns values in 2009. Consequently the period 2006-2009 is more representative of GPD success analysis. As stated earlier the in-sample period between 10.01.2006 and 31.12.2007 represents the start-off basis for GPD calculations, while the out-of-sample period between 01.01.2008 and 31.08.2009 represents the basis for testing the success of the named GPD calculation approaches. According to the results in Table 1, we can see that there is a high range at CROBEX and BUX of 25.54% and 25.83%, while at SBI20 and

BELEXline the range is 15.98% and 16.84% for the daily returns, respectively. According to these results we can conclude that the range in which the changes of returns oscillate in the period 2006-2009 is high.

6. CONCLUDING REMARKS

Risk management has undergone vast changes and gained importance in the last decade due to the increase in the volatility of financial markets, and especially due to the present world economic crisis. This paper has tested the success of the application of the extreme value theory (EVT) in estimating and forecasting the tails of daily return distribution of the analyzed stock indexes in the emerging markets of Serbia, Croatia, Slovenia, and Hungary during the 2006-2009 period. The movements of the returns of the Serbian (BELEXline), Croatian (CROBEX), Slovenian (SBI20), and Hungarian (BUX) stock indexes have been analyzed, i.e. the losses (the left tail) and profits (the right tail) of investment activity. The findings of this research show beyond any doubt the necessity of applying market risk estimation methods, i.e. extreme value theory (EVT), in the framework of a broader analysis of investment processes in emerging markets.

Furthermore the results of this research indicate that the Generalized Pareto Distribution (GPD) fits the tails of the return distribution in selected emerging markets well, and that the daily return distributions have different characteristics at the left and right tails. Namely, risk and reward are not equally likely in these emerging markets. It is clear that emerging markets such as those of the selected emerging markets have unique characteristics, i.e. volatility peculiarities that need to be considered when choosing an adequate approach. There were, however, limitations in the research: the small amount of historical data available, as well as the beginning of the global economic crisis, which had an influence on the stock trade turnover, both globally and on the selected emerging markets. In addition the insufficient liquidity of the selected emerging markets, asymmetrical and low number of trading days with often no transactions at all for several consecutive days, the intense market volatility, and the choice of temporal horizon, are all limiting factors in this research.

The daily returns' considerable oscillations influence the estimation success of the return values of the GPD calculation approaches, i.e. their success testing for the left and right tails (both 2.5% and 5% at the tail). When the GPD calculation is carried out on the limited interval the occurrence of higher daily return values moves the threshold value to the left side for the left tail, and to the right side for the right tail. Due to the shorter period of observation a small number of extremely high daily return values are enough to abruptly change the threshold value. As a consequence a shift of the threshold value occurs (following the change), and so the success of the GPD approach of estimation increases. In GPD calculations using the long interval, the in-sample days with higher daily return values do not result in a considerable change of the threshold value, because the higher number of observation points do not allow abrupt changes in the threshold value. Consequently the GPD calculation of the long interval represents a more rigorous approach, because the oscillations are less after a new day of threshold value calculations is taken into account. For this reason its success is less when compared to GPD calculations with limited interval. The threshold value has high amplitude of changes (is of higher values), especially following a period with great changes (oscillations) of returns.

When daily return oscillations decrease, the success of the GPD calculations using the limited interval is considerable because the threshold value is high, but with the remark that as a result of this on most days the threshold value to a great measure spans the values of the daily returns. Using GPD calculations with the long interval the threshold value is less, because the greater number of calculation values do not allow considerable changes in its value, and as such extreme oscillations of daily returns do not influence the threshold value very much. The threshold value in the long period is more stabile, oscillates less, and the extreme values are in a smaller range. This conditions a less successful result in estimating the values of the returns of the GPD calculation approach, because the days with extreme values of return span the threshold value, but this approach represents a more stabile measure in periods without extreme values, because the estimation is more precise and has less deviation.

The difference that is gained on the basis of GPD calculations with limited and long intervals shows that besides the estimation of the named approaches' success, it is necessary to view the span of the threshold value compared to daily returns. For high threshold values greater success is gained by applying the GPD approach, but in that case the threshold value is much higher than the daily return values in a stable period without high stock index value oscillations in the selected emerging markets. With high threshold values an excessive capital allocation (overestimation of the return) is necessary, which represents a loss in investment processes, and especially so when daily returns are considerably under the threshold value. In practice one hardly knows whether an applied model will under-predict or over-predict the risk in the investment process. According to the results of the research, i.e. testing the performance of the application of the extreme value theory (EVT) on the selected emerging markets, the guidelines for further research should include continuous monitoring of the success of market risk estimations (losses) and profits in investment processes, with special emphasis on the role of optimal threshold determination in increasing the success of estimating the value of returns, and especially so in conditions of global recession.

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APPENDIX

Table A1-1. Threshold value for the in-sample period 2006-2007 (2.5% at the tail)

	Left tail (2.5% at the tail)				Right tail (2.5% at the tail)			
2006-2007	Threshold value	k	σ	μ	Threshold value	k	σ	μ
BELEXline	-2.0428	0.26722	0.32252	0.01535	2.1781	0.10958	0.46997	0.04163
CROBEX	-2.2277	-0.0518	0.65538	0.02687	2.4801	-0.1638	0.8821	0.03786
SBI20	-2.3772	0.19502	0.44038	-0.00105	2.2605	-0.1303	0.7576	0.04165
BUX	-3.4025	-0.1598	1.1954	0.07067	3.1699	-0.17642	1.152	0.04621

Source: Authors' calculations

Table A1-2. Threshold value for the in-sample period 2006-2007 (5% at the tail)

	Left tail (5% at the tail)				Right tail (5% at the tail)			
2006-2007	Threshold value	k	σ	μ	Threshold value	k	σ	μ
BELEX line	-1.4959	0.26722	0.32252	0.01535	1.7081	0.10958	0.46997	0.04163
CROBEX	-1.8456	-0.0518	0.65538	0.02687	2.1263	-0.1638	0.8821	0.03786
SBI20	-1.791	0.19502	0.44038	-0.00105	1.9207	-0.1303	0.7576	0.04165
BUX	-2.9165	-0.1598	1.1954	0.07067	2.7268	-0.17642	1.152	0.04621

Source: Authors' calculations

Table A2-1. Threshold value – limited interval (2.5% at the tail)

		Left tail (2.5% at the tail)				Right tail (2.5% at the tail)			
Observation period	Stock index	Threshold value	k	σ	μ	Threshold value	k	σ	μ
2006-2007	BELEXline	-2.042	0.26722	0.32252	0.01535	2.178	0.10958	0.46997	0.04163
2007-2008	BELEXline	-3.815	0.16183	0.74886	0.03656	3.759	0.20716	0.67893	-9.37E-04
2008-2009	BELEXline	-3.739	-0.00517	0.99776	0.09398	4.537	-0.045	1.3591	-0.08168
2006-2007	CROBEX	-2.227	-0.05175	0.65538	0.02687	2.48	-0.1638	0.8821	0.03786
2007-2008	CROBEX	-6.218	0.16494	1.228	-0.0178	4.578	0.1627	0.89228	0.06759
2008-2009	CROBEX	-6.865	-0.02698	1.9588	-0.01266	6.075	0.04802	1.5022	0.01288
2006-2007	SBI20	-2.377	0.19502	0.44038	-0.00105	2.26	-0.1303	0.7576	0.04165
2007-2008	SBI20	-5.205	0.11832	1.1149	0.04917	3.797	0.06535	0.88983	0.08568
2008-2009	SBI20	-5.181	0.13544	1.0574	0.2117	4.341	0.07601	1.0086	0.04646
2006-2007	BUX	-3.402	-0.15979	1.1954	0.07067	3.169	-0.17642	1.152	0.04621
2007-2008	BUX	-5.746	0.17435	1.082	0.14574	5.225	0.21571	0.91203	0.08354
2008-2009	BUX	-6.563	-0.00931	1.7755	0.1254	6.971	-0.00602	1.9094	0.00504

Source: Authors' calculations

Table A2-2. Threshold value – limited interval (5% at the tail)

		Left tail (5% at the tail)				Right tail (5% at the tail)			
Observation period	Stock index	Threshold value	k	σ	μ	Threshold value	k	σ	μ
2006-2007	BELEXline	-1.4959	0.26722	0.32252	0.01535	1.708	0.10958	0.46997	0.04163
2007-2008	BELEXline	-2.9234	0.16183	0.74886	0.03656	2.817	0.20716	0.67893	-9.37E-04
2008-2009	BELEXline	-3.06	-0.0052	0.99776	0.09398	3.727	-0.045	1.3591	-0.08168
2006-2007	CROBEX	-1.8456	-0.0518	0.65538	0.02687	2.126	-0.1638	0.8821	0.03786
2007-2008	CROBEX	-4.74	0.16494	1.228	-0.0178	3.512	0.1627	0.89228	0.06759
2008-2009	CROBEX	-5.6245	-0.027	1.9588	-0.01266	4.852	0.04802	1.5022	0.01288
2006-2007	SBI20	-1.791	0.19502	0.44038	-0.00105	1.92	-0.1303	0.7576	0.04165
2007-2008	SBI20	-4.0576	0.11832	1.1149	0.04917	3.03	0.06535	0.88983	0.08568
2008-2009	SBI20	-4.0285	0.13544	1.0574	0.2117	3.439	0.07601	1.0086	0.04646
2006-2007	BUX	-2.9165	-0.1598	1.1954	0.07067	2.726	-0.17642	1.152	0.04621
2007-2008	BUX	-4.4024	0.17435	1.082	0.14574	3.923	0.21571	0.91203	0.08354
2008-2009	BUX	-5.3708	-0.0093	1.7755	0.1254	5.673	-0.00602	1.9094	0.00504

Source: Authors' calculations

Table A3-1. Threshold value – long interval (2.5% at the tail)

		Left tail (2.5% at the tail)				Right tail (2.5% at the tail)			
Stock index	Observation period	Threshold value	k	σ	μ	Threshold value	k	σ	μ
BELEXline	2006	-0.875	-0.1096	0.28794	0.00185	1.007	-0.39508	0.50691	0.02295
	2006-2007	-2.042	0.26722	0.32252	0.01535	2.178	0.10958	0.46997	0.04163
	2006-2008	-3.38	0.23651	0.57231	0.01017	2.923	0.24982	0.47764	0.03002
	2006-2009	-3.297	0.12302	0.70525	0.00489	3.383	0.17726	0.64871	0.0059
CROBEX	2006	-1.644	-0.2112	0.63394	0.02039	2.202	-0.18916	0.82451	0.01287
	2006-2007	-2.227	-0.0518	0.65538	0.02687	2.48	-0.1638	0.8821	0.03786
	2006-2008	-5.346	0.24863	0.88311	0.01089	3.835	0.12008	0.81353	0.05411
	2006-2009	-5.57	0.18841	1.0449	0.00422	4.376	0.10414	0.96318	0.04492
SBI20	2006	-1.333	0.15905	0.26379	0.01004	1.846	-0.01002	0.4990.3	0.03941
	2006-2007	-2.377	0.19502	0.44038	-0.00105	2.26	-0.1303	0.7576	0.04165
	2006-2008	-4.536	0.23062	0.78265	-0.01555	3.303	0.09597	0.73415	0.05401
	2006-2009	-4.182	0.18929	0.78247	0.00622	3.225	0.04545	0.79027	0.05116
BUX	2006	-3.78	-0.2216	1.4978	0.00587	3.404	-0.26676	1.4239	0.06156
	2006-2007	-3.402	-0.1598	1.1954	0.07067	3.169	-0.17642	1.152	0.04621
	2006-2008	-5.168	0.08611	1.1646	0.11151	4.639	0.0934	1.0343	0.0843
	2006-2009	-5.243	0.00485	1.3839	0.09262	5.31	0.06294	1.2655	0.05539

Source: Authors' calculations

Table A3-2. Threshold value – long interval (5% at the tail)

		Left tail (5% at the tail)				Right tail (5% at the tail)			
Stock index	Observation period	Threshold value	k	σ	μ	Threshold value	k	σ	μ
BELEXline	2006	-0.737	-0.1096	0.28794	0.00185	0.913	-0.39508	0.50691	0.02295
	2006-2007	-1.495	0.26722	0.32252	0.01535	1.708	0.10958	0.46997	0.04163
	2006-2008	-2.505	0.23651	0.57231	0.01017	2.159	0.24982	0.47764	0.03002
	2006-2009	-2.559	0.12302	0.70525	0.00489	2.57	0.17726	0.64871	0.0059
CROBEX	2006	-1.427	-0.2112	0.63394	0.02039	1.898	-0.18916	0.82451	0.01287
	2006-2007	-1.845	-0.0518	0.65538	0.02687	2.126	-0.1638	0.8821	0.03786
	2006-2008	-3.939	0.24863	0.88311	0.01089	2.991	0.12008	0.81353	0.05411
	2006-2009	-4.21	0.18841	1.0449	0.00422	3.431	0.10414	0.96318	0.04492
SBI20	2006	-1.022	0.15905	0.26379	0.01004	1.512	-0.01002	0.4990.3	0.03941
	2006-2007	-1.791	0.19502	0.44038	-0.00105	1.92	-0.1303	0.7576	0.04165
	2006-2008	-3.362	0.23062	0.78265	-0.01555	2.602	0.09597	0.73415	0.05401
	2006-2009	-3.16	0.18929	0.78247	0.00622	2.587	0.04545	0.79027	0.05116
BUX	2006	-3.284	-0.2216	1.4978	0.00587	2.998	-0.26676	1.4239	0.06156
	2006-2007	-2.916	-0.1598	1.1954	0.07067	2.726	-0.17642	1.152	0.04621
	2006-2008	-4.091	0.08611	1.1646	0.11151	3.659	0.0934	1.0343	0.0843
	2006-2009	-4.268	0.00485	1.3839	0.09262	4.227	0.06294	1.2655	0.05539

Source: Authors' calculations

Table A4-1. Threshold value for the left tail for each tested index (2.5% at the tail)

Stock index	Left tail	2006-2007	2006-2008	2006-2009
BELEXline	Limited interval	-2.042	-3.815	-3.739
	Long interval	-2.042	-3.38	-3.297
CROBEX	Limited interval	-2.227	-6.218	-6.865
	Long interval	-2.227	-5.346	-5.570
SBI20	Limited interval	-2.377	-5.205	-5.181
	Long interval	-2.377	-4.536	-4.182
BUX	Limited interval	-3.402	-5.746	-6.563
	Long interval	-3.402	-5.168	-5.243

Source: Authors' calculations

Table A4-2. Threshold value for the left tail for each tested stock index (5% at the tail)

Stock index	Left tail	2006-2007	2006-2008	2006-2009
BELEXline	Limited interval	-1.4959	-2.9234	-3.06
	Long interval	-1.4959	-2.505	-2.5595
CROBEX	Limited interval	1.8456	-4.74	-5.6245
	Long interval	-1.8456	-3.9396	-4.2104
SBI20	Limited interval	-1.791	-4.0576	-4.0285
	Long interval	-1.791	-3.3627	-3.1606
BUX	Limited interval	-2.9165	-4.4024	-5.3708
	Long interval	-2.9165	-4.0917	-4.2687

Source: Authors' calculations

Table A4-3. Threshold value for the right tail for each tested stock index (2.5% at the tail)

Stock index	Right tail	2006-2007	2006-2008	2006-2009
BELEXline	Limited interval	2.178	3.759	4.537
	Long interval	2.178	2.923	3.383
CROBEX	Limited interval	2.48	4.578	6.075
	Long interval	2.48	3.835	4.376
SBI20	Limited interval	2.26	3.797	4.341
	Long interval	2.26	3.303	3.225
BUX	Limited interval	3.169	5.225	6.971
	Long interval	3.169	4.639	5.31

Source: Authors' calculations

Table A4-4. Threshold value for the right tail for each tested stock index (5% at the tail)

Stock index	Right tail	2006-2007	2006-2008	2006-2009
BELEXline	Limited interval	1.7081	2.8177	3.7273
	Long interval	1.7081	2.15952	2.5701
CROBEX	Limited interval	2.1263	3.5125	4.8529
	Long interval	2.1263	2.991	3.4311
SBI20	Limited interval	1.9207	3.0302	3.4396
	Long interval	1.9207	2.6021	2.5873
BUX	Limited interval	2.7268	3.9238	5.6738
	Long interval	2.7268	3.6597	4.2275

Source: Authors' calculations

Table A5. Performance testing of the GPD application for BELEXline in 2008 – limited interval

BELEX line	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	157	90.75	153	88.95	71	91.03	70	89.74
Unsuccessful	16	9.25	19	11.05	7	8.97	8	10.26
Total	173	68.92	172	68.53	78	31.08	78	31.08

Source: Authors' calculations

Table A6. Performance testing of the GPD application for BELEXline in 2008 – long interval

BELEX line	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	155	89.60	149	86.63	71	91.03	69	87.34
Unsuccessful	18	10.40	23	13.37	7	8.97	10	12.66
Total	173	68.92	172	68.53	78	31.08	79	31.47

Source: Authors' calculations

Table A7. Performance testing of the GPD application for BELEXline in 2009 – limited interval

BELEX line	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	92	100.00	91	98.91	71	97.26	65	90.28
Unsuccessful	0	0.00	1	1.09	2	2.74	7	9.72
Total	92	55.76	92	55.76	73	44.24	72	43.64

Source: Authors' calculations

Table A8. Performance testing of the GPD application for BELEXline in 2009 – long interval

BELEX line	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	91	98.91	90	97.83	69	94.52	58	79.45
Unsuccessful	1	1.09	2	2.17	4	5.48	15	20.55
Total	92	55.76	92	55.76	73	44.24	73	44.24

Source: Authors' calculations

Table A9. Performance testing of the GPD application for CROBEX in 2008 – limited interval

CROBEX	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	120	86.96	104	75.36	102	91.89	97	87.39
Unsuccessful	18	13.04	34	24.64	9	8.11	14	12.61
Total	138	55.20	138	55.42	111	44.40	111	44.58

Source: Authors' calculations

Table A10. Performance testing of the GPD application for CROBEX in 2008 – long interval

CROBEX	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	113	81.88	99	71.74	102	91.89	95	85.59
Unsuccessful	25	18.12	39	28.26	9	8.11	16	14.41
Total	138	55.42	138	55.42	111	44.58	111	44.58

Source: Authors' calculations

Table A11. Performance testing of the GPD application for CROBEX in 2009 – limited interval

CROBEX	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	77	98.72	76	97.44	82	95.35	76	88.37
Unsuccessful	1	1.28	2	2.56	4	4.65	10	11.63
Total	78	47.56	78	47.56	86	52.44	86	52.44

Table A12. Performance testing of the GPD application for CROBEX in 2009 – long interval

CROBEX	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	77	98.72	72	92.31	78	90.70	70	81.40
Unsuccessful	1	1.28	6	7.69	8	9.30	16	18.60
Total	78	47.56	78	47.56	86	52.44	86	52.44

Source: Authors' calculations

Table A13. Performance testing of the GPD application for SBI20 in 2008 – limited interval

SBI20	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	129	87.16	121	81.21	83	85.57	82	84.54
Unsuccessful	19	12.84	28	18.79	14	14.43	15	15.46
Total	148	60.16	149	60.57	97	39.43	97	39.43

Source: Authors' calculations

Table A14. Performance testing of the GPD application for SBI20 in 2008 – long interval

SBI20	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	128	86.49	118	79.19	84	86.60	82	84.54
Unsuccessful	20	13.51	31	20.81	13	13.40	15	15.46
Total	148	60.16	149	60.57	97	39.43	97	39.43

Source: Authors' calculations

Table A15. Performance testing of the GPD application for SBI20 in 2009 – limited interval

SBI20	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	81	100.00	81	100.00	83	97.65	81	95.29
Unsuccessful	0	0.00	0	0.00	2	2.35	4	4.71
Total	81	48.80	81	48.80	85	51.20	85	51.20

Source: Authors' calculations

Table A16. Performance testing of the GPD application for SBI20 in 2009 – long interval

SBI20	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	81	100.00	80	98.77	83	97.65	81	95.29
Unsuccessful	0	0.00	1	1.23	2	2.35	4	4.71
Total	81	48.80	81	48.80	85	51.20	85	51.20

Source: Authors' calculations

Table A17. Performance testing of the GPD application for BUX in 2008 – limited interval

BUX	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	125	88.65	117	82.98	95	87.16	92	84.40
Unsuccessful	16	11.35	24	17.02	14	12.84	17	15.60
Total	141	56.40	141	56.40	109	43.60	109	43.60

Source: Authors' calculations

Table A18. Performance testing of the GPD application for BUX in 2008 – long interval

BUX	Left tail				Right tail			
2008	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	128	90.78	120	85.11	95	87.16	92	84.40
Unsuccessful	13	9.22	21	14.89	14	12.84	17	15.60
Total	141	56.40	141	56.40	109	43.60	109	43.60

Source: Authors' calculations

Table A19. Performance testing of the GPD application for BUX in 2009 – limited interval

BUX	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	74	98.67	73	97.33	84	93.33	75	83.33
Unsuccessful	1	1.33	2	2.67	6	6.67	15	16.67
Total	75	45.45	75	45.45	90	54.55	90	54.55

Source: Authors' calculations

Table A20. Performance testing of the GPD application for BUX in 2009 – long interval

BUX	Left tail				Right tail			
2009	2.50%	Difference in %	5%	Difference in %	2.50%	Difference in %	5%	Difference in %
Successful	74	98.67	73	97.33	81	90.00	75	83.33
Unsuccessful	1	1.33	2	2.67	9	10.00	15	16.67
Total	75	45.45	75	45.45	90	54.55	90	54.55

Source: Authors' calculations

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