ECONOMIC ANNALS, Volume LIV No. 182 / July – September 2009
UDC: 3.33 ISSN: 0013-3264

COMMUNICATIONS

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DOI:10.2298/EKA0982119G

CONSTRAINTS ON CREDIT, CONSUMER BEHAVIOUR AND THE DYNAMICS OF WEALTH

ABSTRACT: *This paper develops a simple macroeconomic model where the pattern of wealth accumulation is determined by a credit multiplier and the way households react to short-term fluctuations. Given this setup, long term wealth dynamics are eventually characterized by the presence of endogenous cycles.*

KEY WORDS: *credit constraints, financial development, consumer confidence, endogenous business cycles, nonlinear dynamics.*

JEL CLASSIFICATION: O41, E32, C61

* Escola Superior de Comunicação Social, Lisbon, Portugal. Acknowledgements: Financial support from the Fundação Ciência e Tecnologia, Lisbon, is gratefully acknowledged, under the contract No POCTI/ECO/48628/2002, partially funded by the European Regional Development Fund (ERDF). I also thank the helpful comments and suggestions of two anonymous referees; the usual disclaimer applies.

1. INTRODUCTION

The relationship between collateral requirements or credit constraints and business cycles is a relevant theme for discussion in current macroeconomic analysis. This subject gained visibility with the contributions of Bernanke and Gertler (1989) and Kyotaki and Moore (1997), and was further developed through the work of Aghion, Bacchetta and Banerjee (2001, 2004), Demirguç-Kunt and Levine (2001), Amable, Chatelain and Ralf (2004), Aghion, Angeletos, Banerjee and Manova (2005) and Caballé, Jarque and Michetti (2006), among others. The essential conclusion from these studies is that markets where firms face some degree of credit constraint are markets where investment is strongly pro-cyclical, thus the main economic aggregates will be subject to amplified volatility that tends to persist over time. Basically, the lesson drawn from this literature is that cycles are more likely to be observed for some specific levels of financial development than for others.[1] For instance, Caballé, Jarque and Michetti (2006), hereafter CJM, concluded that stability characterizes low and high levels of financial development, while for intermediate levels endogenous business cycles dominate.

In this note, we use the CJM model to present an alternative approach to the formation of endogenous cycles for intermediate levels of financial development. We consider the same scenario as the previous authors, but with two important changes: firstly, physical capital represents the unique production input (we ignore the country-specific input with a constant supply assumed in the referred model); secondly, we introduce a mechanism through which households respond to short-term wealth deviations from a potential wealth level. Furthermore, the analysis is undertaken under an endogenous growth framework; this is translated in the assumption of a constant marginal returns AK production function.

This note is organized as follows. Section 2 is a brief description of types of bifurcations in two dimensional systems. In the specific model to be developed, endogenous fluctuations arise as the result of a particular type of bifurcation. In order to understand its specificity and implications, it is neccessary to provide a short guide to local bifurcations capable of generating complex global dynamics. Section 3 develops the model's structure. This is a conventional growth model involving a financial market where credit constraints are established. Sections 4 and 5 characterize the dynamics of the model, the first in terms of local

1 The level of financial development is translated on the degree of constraints to credit.

stability results and the second with respect to global dynamics and endogenous fluctuations. Section 5 contains conclusions and policy implications.

2. BIFURCATIONS IN TWO-DIMENSIONAL SYSTEMS: A BRIEF GUIDE

The model in the following sections comprises a two-dimensional system from which endogenous fluctuations arise under particular circumstances. In generic terms, periodic and aperiodic cycles eventually emerge when a bifurcation takes place. A bifurcation occurs at the point where the topological properties of the system are changed (e.g. the point where stability is lost). Typically, in a low-dimensional context such as the one under consideration, the bifurcation separates a region of fixed-point stability (where convergence from a given initial point towards a unique equilibrium is observed) from a region where locally the system is unstable.[2] In a global dynamics perspective this second region may reveal the presence of a sub-area of cycles of any periodicity (or completely aperiodic cycles) before instability (divergence from the steady state) effectively sets in.

The different kinds of bifurcations observable in two-dimensional dynamic systems are briefly listed below. For a detailed study of local bifurcations and their implications for global dynamic behaviour see, e.g. Medio and Lines (2001) or Grandmont (2008). Classification of different types of bifurcations is possible after computing the Jacobian matrix of the model's dynamic system. In two-dimensional systems, the Jacobian matrix is a 2×2 matrix with its elements corresponding to the derivative of each equation with respect to each endogenous variable (and these derivatives are evaluated in the steady state). We will denote this matrix by the letter J.

Knowledge of the trace and the determinant of J constitutes the necessary information to identify the types of bifurcation a system can eventually go through. Stability holds if the following conditions are simultaneously satisfied:

$$1+Tr(J)+Det(J)>0;$$
$$1-Tr(J)+Det(J)>0;$$
$$1-Det(J)>0$$

2 Local instability may involve the two dimensions of the system or only one. In the second case, the system is saddle-path stable.

If the above set of inequalities holds, then the two eigenvalues of J lie inside the unit circle, a condition that effectively guarantees stability. Stability is lost (i.e., a bifurcation occurs) when one of the eigenvalues crosses the unit circle, i.e. when a change in the value of a selected parameter triggers a bifurcation. Violations of the unit circle condition imply that one or more of the above trace-determinant positive relations are lost. Bifurcations can be classified as follows:

(i) Let the second and third stability conditions hold. When $1+Tr(J)+Det(J)$ becomes equal to zero, one of the eigenvalues will assume the value -1. In this case, a *flip bifurcation* occurs. In non-linear systems, the flip bifurcation frequently produces a period-doubling route to chaotic motion.

(ii) Let the first and the third stability conditions hold. Now consider that $1-Tr(J)+Det(J)=0$. in which case one of the eigenvalues touches the upper bound of the unit circle. This scenario may lead to a *fold bifurcation* or a *transcritical bifurcation* or a *pitchfork bifurcation*. These types of bifurcations may also culminate in totally aperiodic time series, for instance through intermittency (an intermittent system is a system that suffers infrequent variations of large amplitude).

(iii) When the third stability condition fails to hold, the bifurcation is of the Neimark-Sacker type. A *Neimark-Sacker bifurcation* (or Hopf-bifurcation in discrete time) occurs when the modulus of a pair of complex conjugate eigenvalues assumes the value one. In a two-dimensional system, the modulus of two complex eigenvalues corresponds to the determinant of the assumed matrix. This type of bifurcation tends to imply a sudden jump from the region of stability into an area, in the space of parameters, where quasi-periodic cycles emerge. Such cycles commonly degenerate into a region of chaos.

Chaotic motion is here associated with the observation of sensitive dependence on initial conditions, i.e., no matter how close the initial points of two chaotic series generated by the same system are, they will follow completely divergent orbits. Although we are referring to exclusively deterministic time series, no pattern can be predicted or inferred from one numerical concretization of the system to another.

It is the last type of bifurcation that we will find in the current model. The generated endogenous business cycles will be quasi-periodic cycles for some combinations of parameter values, , and chaotic fluctuations for other values,. The type of produced cycles represent a kind of time series that have an admissible pattern in respect of the characterization of the evolution of economic aggregates over time, because they allow for some smoothness and persistence in the evolution of variables. Phases in which the value of the assumed variable falls and phases in which it increases tend to last for several time periods.

3. THE STRUCTURE OF THE MODEL

Consider a competitive economy populated by a large number of households and firms. Firms produce a tradable good under an AK production function, $y_t=Ak_t$, with $A>0$ a technology index and k_t, y_t the per capita levels of physical capital and output in moment t.[3] We assume that capital fully depreciates after one period, hence $i_t=k_t$, with i_t per capita investment. Households have the possibility to lend their financial resources directly to firms if the marginal productivity of capital (A) is above the economy's nominal interest rate (r); hereafter, we impose this constraint on parameters: $A>r$.

If the credit market is subject to some kind of imperfection, firms' financial resources (that we designate by wealth) will serve as collateral for the loans, and thus firms cannot borrow an amount over μw_t, with w_t the level of per capita wealth and μ a credit multiplier that reflects the degree of financial development of the economy. Households and firms agree on applying to the productive projects the largest amount of credit that can be subject to transaction, and thus investment in moment t corresponds to $i_t=(1+\mu)\cdot w_t$. Finally, the structure of the model is completed with a difference equation reflecting wealth dynamics,

$$w_{t+1} = y_t - r\mu w_t - c_t,\ w_0 \text{ given.} \tag{1}$$

Equation (1) states that wealth in moment $t+1$ corresponds to income in t, less the cost of debt and less the resources allocated to consumption (c_t is per capita consumption). In the CJM model, c_t corresponds to a constant fraction of income less debt payment. We generalize this assumption by considering that the marginal propensity to consume depends on the observable difference between effective levels of wealth and expected or potential wealth. In practice, agents react to business cycles by adopting the following rule: the higher the level of the last period's wealth relative to the benchmark level of wealth, the more optimistic households will be and, accordingly, the higher will be the share of consumption out of income. In other words, low (high) levels of observed accumulated wealth

3 To simplify, assume that population does not grow.

relative to a benchmark level will imply a precautionary (confident) behaviour that is translated into a higher (lower) savings rate.

Formally, we consider $c_t = c \cdot (y_t - r\mu w_t)\, g(w_{t-1})$, $c \in (0,1)$ and $g(w_t)$ a positive, continuous and differentiable function, with $g' > 0$. Consider w_t^* as the potential level of wealth; this is supposed to represent a wealth trend that grows at the same rate, γ, for all t. Thus, function g will be such that $g(w_t)\,|_{w_t = w_t^*} = 1$, $g(w_t)\,|_{w_t > w_t^*} > 1$ and $g(w_t)\,|_{w_t < w_t^*} < 1$. The following functional form fulfils the required properties: $g(w_t) = \left(\frac{w_t}{w_t^*}\right)^a$, $a > 0$.

The reduced form of the dynamic system is straightforward to obtain given the previous information,

$$w_{t+1} = \left[A + (A - r)\cdot\mu\right]\cdot\left[1 - c\cdot\left(\frac{w_{t-1}}{w_{t-1}^*}\right)^a\right]\cdot w_t \tag{2}$$

Because production is subject to constant marginal returns, all relevant variables (k_t, y_t, i_t, c_t and w_t) grow at a constant positive rate in the steady state. Let this rate be γ, and thus we define variable $\hat{w}_t \equiv \frac{w_t}{(1+\gamma)^t}$ and constant $\hat{w}^* \equiv \frac{w_t^*}{(1+\gamma)^t}$. In the steady state, effective wealth grows at a same rate as potential wealth. However, before this long-run result is eventually attained, the growth rates might differ. Rewriting (2),

$$\hat{w}_{t+1} = \frac{A + (A - r)\cdot\mu}{1+\gamma}\cdot\left[1 - c\cdot\left(\frac{\hat{w}_{t-1}}{\hat{w}^*}\right)^a\right]\cdot\hat{w}_t \tag{3}$$

Equation (3) has a unique equilibrium point: $\overline{w} = \left[\frac{1}{c}\cdot\left(1 - \frac{1+\gamma}{A + (A - r)\cdot\mu}\right)\right]^{1/a}\cdot\hat{w}^*$.

Note, relative to the steady state value, to guarantee $\overline{w} > 0$, the following inequality must hold: $A + (A - r)\cdot\mu > 1 + \gamma$. This condition indicates that there is an upper bound on how much the economy can grow. If the constant steady state growth rate γ passes a given threshold then no meaningful equilibrium is encountered for the wealth variable. A higher rate γ is admissible for larger values of the technology index and of the credit multiplier and for a lower interest rate. Another way to look at the condition is that it imposes a floor on the value of the credit multiplier: $\mu > \frac{1+\gamma-A}{A-r}$ is the minimal requirement in terms of credit availability for the economy to be able to accumulate wealth.

A first relevant result is expressed in the following proposition,

Proposition 1: *The wealth model, with credit constraints and consumption reaction to deviations from the last period's potential wealth, reveals that the higher the level of financial development of the economy, the larger the amount of accumulated wealth, in the steady state.*

Proof: Take the steady state expression for the wealth variable and compute derivative $\overline{w}_{\mu}=\frac{\partial\overline{w}}{\partial\mu}$. The computation gives,

$\overline{w}_{\mu}=\frac{\hat{w}^{*}}{a}\cdot\left[\frac{1}{c}\cdot\left(1-\frac{1+\gamma}{A+(A-r)\cdot\mu}\right)\right]^{\frac{1-a}{a}}\cdot\left[\frac{1}{c}\cdot\frac{(1+\gamma)\cdot(A-r)}{\left[A+(A-r)\cdot\mu\right]^{2}}\right]$. Because this is a positive value, one infers that the accumulated level of wealth is positively correlated with financial development (measured by the credit multiplier parameter, μ)□

4. LOCAL DYNAMICS

In this section, we address the dynamics of equation (3) in the neighbourhood of point $\overline{w}$. This requires defining variable $\tilde{w}_t\equiv\hat{w}_t+\overline{w}$. With this variable, we turn equation (3) into a two equation system with two endogenous variables and just one time lag,

$$\begin{cases}\tilde{w}_{t+1}=\frac{A+(A-r)\cdot\mu}{1+\gamma}\cdot\left[1-c\cdot\left(\frac{\tilde{z}_t+\overline{w}}{\hat{w}^{*}}\right)^{a}\right]\cdot(\tilde{w}_t+\overline{w})-\overline{w}\\ \tilde{z}_{t+1}=\tilde{w}_t\end{cases}\qquad(4)$$

The presentation of system (4) requires consideration of the auxiliary variable $\tilde{z}_t\equiv\tilde{w}_{t-1}$.

Around the balanced growth path, system (4) takes the linearized form

$$\begin{bmatrix}\tilde{w}_{t+1}\\ \tilde{z}_{t+1}\end{bmatrix}=\begin{bmatrix}1 & -a\cdot\frac{A+(A-r)\cdot\mu-(1+\gamma)}{1+\gamma}\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}\tilde{w}_t\\ \tilde{z}_t\end{bmatrix}\qquad(5)$$

Take into consideration that the steady state values of variables $\tilde{w}_t$ and $\tilde{z}_t$ are, in both cases, 0. Proposition 2 synthesizes the local dynamics result.

Proposition 2*: The wealth model under analysis is locally stable for* $\mu \in \left(\frac{1+\gamma-A}{A-r}; \frac{\left(\frac{1+a}{a}\right)\cdot(1+\gamma)-A}{A-r} \right)$*; when* $\mu = \frac{\left(\frac{1+a}{a}\right)\cdot(1+\gamma)-A}{A-r}$*, the system undergoes a Neimark-Sacker bifurcation.*

Proof: The Jacobian matrix in (5) has a positive determinant, $Det(J) = a \cdot \frac{A+(A-r)\cdot\mu-(1+\gamma)}{1+\gamma}$, and its trace is *Tr*(*J*)=1. Thus, stability conditions 1-*Tr*(*J*)+*Det*(*J*)>0 and 1+*Tr*(*J*)+*Det*(*J*)>0 are always satisfied. The only possible bifurcation occurs when the eigenvalues of the matrix are a pair of complex conjugate values with modulus equal to one, which is equivalent to saying that *Det*(*J*)=1. The equality expression in the proposition is determined by solving this last condition in order to μ. Stability requires 1-*Det*(*J*)>0☐

From proposition 1, we have concluded that the less constrained credit is, the larger is the amount of wealth the economy accumulates in the long run, while from proposition 2 one observes that there is a stability ceiling. If freedom to offer credit is too high, the guarantee that the steady-state level of wealth is achieved vanishes. Therefore, one can interpret this theoretical structure as indicating the advantages of both financial development and financial responsibility, in the sense that excessive credit may disrupt the financial system as agents fail to pay back the large amounts of resources they have borrowed.

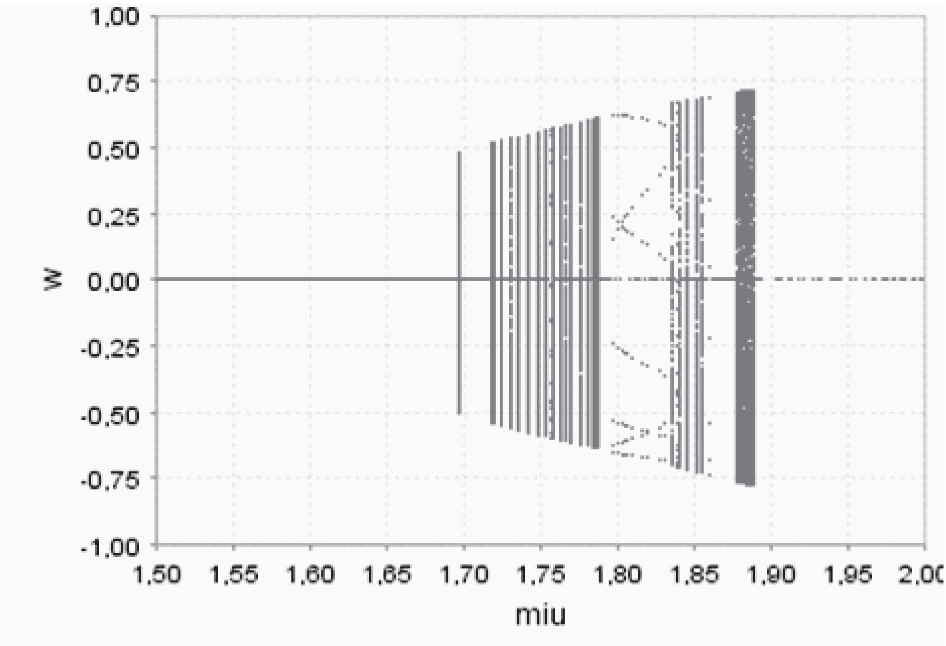
Local dynamics conceal meaningful features of the model. Firstly, cycles appear to be absent. The theory of nonlinear dynamics points to the eventual presence of cycles after a bifurcation. In our concrete system, we should expect to effectively encounter a fixed point in the stability area identified in proposition 2, and aperiodic motion after the bifurcation and before instability truly sets in. This becomes evident in the global analysis of the following section. Secondly, the global analysis of this specific model shows that some points of stability are present in the locally unstable area, a result that can be used to justify a conclusion similar to the one in the CJM model: financial instability (cycles) occurs for intermediate levels of development, while stability characterizes low and high values of market development.

5. GLOBAL DYNAMICS

To address global dynamics consider an array of reasonable parameter values:[4] [$A; c; \gamma; r; w^*; a$]=[1; 0.75; 0.04; 0.03; 1; 0.7] and let μ be the bifurcation parameter. In this particular case, the system is stable for 0.041<μ<1.573.

Figure 1 is the bifurcation diagram. A bifurcation, that occurs for μ=1.573, separates an area of stability from an area where invariant cycles can be observed. A region where endogenous cycles occur, is followed by one where stability and instability alternate. Recall that $\tilde{w}_t$ is a variable that was modified twice. First it was detrended, and then it was adjusted to obtain a balanced growth path where the variable takes the value zero. In the long run, the original variable w_t grows exponentially, with a detrended value equal to $\overline{w}$.

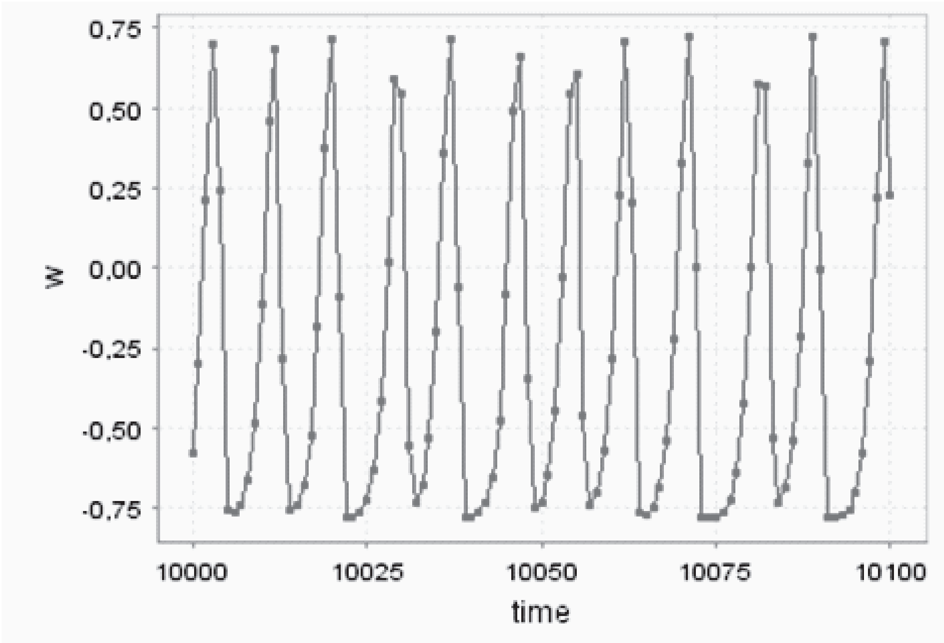
Figure 1 – Bifurcation diagram (1.5<μ<2).



4 In particular, observe that the assumed marginal propensity to consume corresponds to 75% of income, that the growth rate is considered to be 4% and that the interest rate takes the value 3%.

To demonstrate that this framework produces everlasting endogenous business cycles for specific values of the credit parameter, we present a time series of $\tilde{w}_t$ in figure 2. The Neimark-Sacker bifurcation, or Hopf bifurcation in discrete time, is able to generate the kind of dynamics that characterize real-world business cycles, in the sense that several consecutive periods of increasing wealth are followed by periods where there is a slowdown in the growth of wealth. Another relevant feature observable in the displayed figure is that phases of falling detrended wealth typically last fewer periods (2 to 3 periods) than phases of recovery (5 to 6 periods).

Figure 2 – Time series (*μ*=1.891).



Taking the same value of μ as in figure 2, we draw an attractor that reveals the long-run relationship between $\tilde{w}_{t-1}$ and $\tilde{w}$ (figure 3), and the basin of attraction that furnishes the set of initial points that allow for a convergence towards the long-run attractor (figure 4).

Figure 3 – Attracting set (μ=1.891).

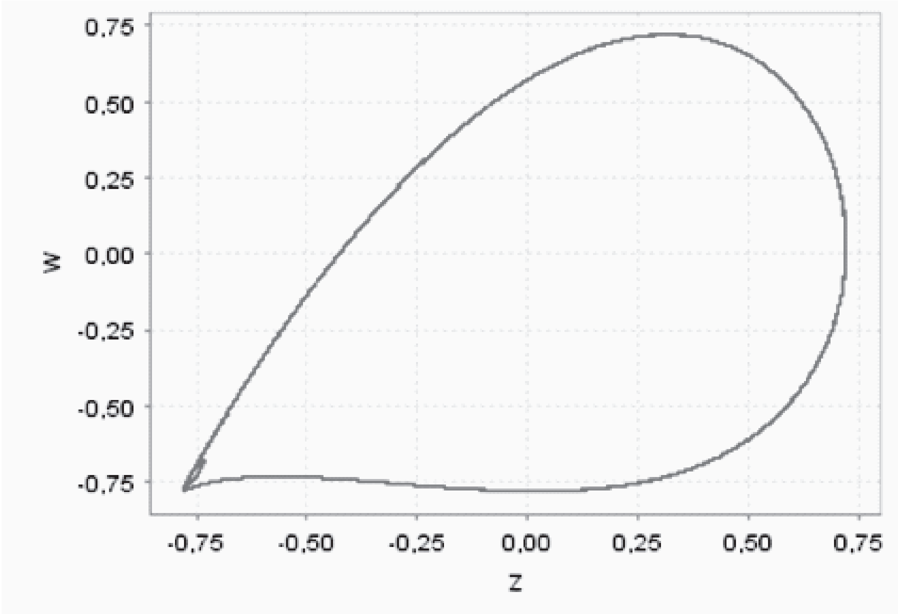
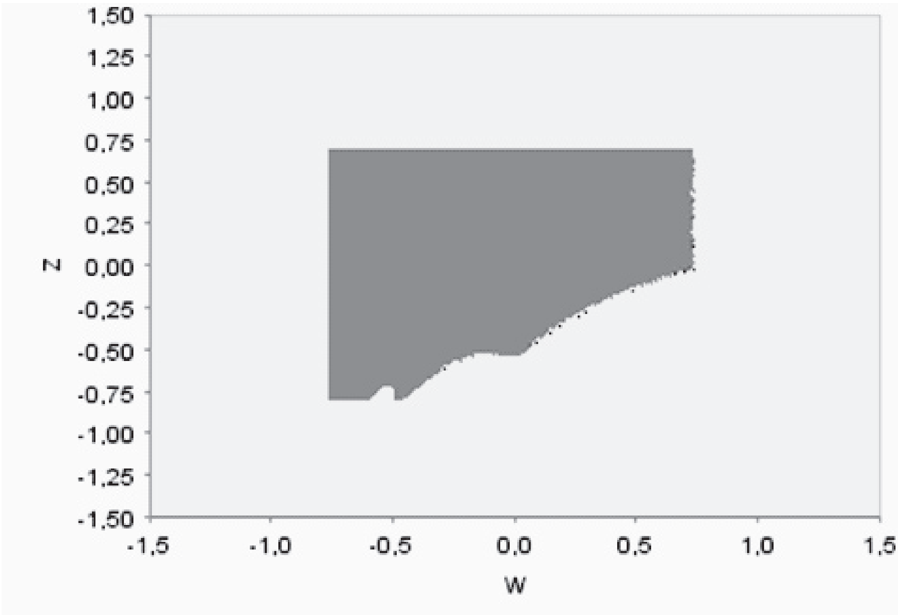


Figure 4 – Basin of attraction (μ=1.891).



6. CONCLUSIONS AND POLICY IMPLICATIONS

Following the literature on credit constraints and business cycles, we have considered a basic AK endogenous growth model, where financial development is addressed through a credit multiplier and where economic agents take consumption decisions by weighing the last period's difference between observed and potential wealth (if this difference is positive, consumer optimism rises and there is an increase in the share of income spent; with a negative difference, households become more cautious and prefer to increase the marginal propensity to save).

The proposed model is able to demonstrate two important points:

i) As the level of financial development level rises, per capita wealth, in the steady state, also increases;
ii) Fixed point stability is found for low levels of financial development (and thus low levels of accumulated wealth). An intermediate level of the credit multiplier allows identification of aperiodic cycles generated through a Neimark-Sacker bifurcation. High levels of financial development are characterized by alternation of stability and instability.

The instability resulting from high levels of the credit multiplier adds a new feature to the CJM model: high credit multipliers, typical of loans with no collateral, imply a high risk in the credit market, in the sense that borrowers may not be able to repay their debts. In this case, instability may be interpreted as a financial crisis that imposes the need to substantially reduce the level of available credit.

These results of modelling a growing economy provide relevant policy implications on how public authorities (and, in particular, central banks) should regulate the financial system. If one accepts the view that developed economies work in a similar fashion to the one described (which assumes that consumer sentiment decisively influences the allocation of household disposable income and that economies tend to grow at a positive constant rate at the potential steady state), the way in which the availability of credit is regulated is critical to maintain a stable economy. Relatively strict controls on credit availability guarantee stability but provide a relatively low steady-state wealth level. Looser constraints on credit can provide higher levels of wealth, but at the cost of a larger risk of sudden and undesirable fluctuations in economic activity. Under this interpretation, severe recessions may be the result of excessive credit availability, thus implying central banks have a fundamental role in managing business fluctuations.

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