

*Ana Manola\* and Branko Urošević\*\**

## OPTION-BASED VALUATION OF MORTGAGE-BACKED SECURITIES

**ABSTRACT:** *Pure econometric approaches to pricing mortgage-backed securities (MBSs) - principal pricing vehicles used by financial practitioners - fail to capture their true risks. This point was powerfully driven home by the global financial crisis. Since prior to the crisis default rates of MBSs were quite modest, econometric pricing models systematically underestimated the possibility of default. As a result, MBSs were severely overvalued. It is widely believed that the global crisis was largely triggered by incorrect valuation of mortgage-backed securities. In the aftermath, it is important to revisit the foundations for pricing*

*MBSs and to pay much closer attention to default risk. This paper introduces a comprehensive model for valuation of fixed-rate pass-through mortgage-backed securities in a simple option-based framework. In the model, we use bivariate binomial tree approach to simultaneously model prepayment and default options. Our simulation results demonstrate that the proposed model has sufficient flexibility to capture the two principal risks.*

**KEY WORDS:** *mortgage-backed security (MBS), prepayment, default, bivariate binomial pricing technique*

**JEL CLASSIFICATION:** C02, C60, G10, G12, G21

---

\* Economic and Commercial Office of the Embassy of Spain, Belgrade

\*\* Faculty of Economics, University of Belgrade, urosevic@ekof.bg.ac.rs. Branko Urošević gratefully acknowledges support by the Serbian Ministry of Science and Technology, Grant No 149041.

## 1. INTRODUCTION

Mispricing of mortgage-backed securities (MBS) has been widely blamed for triggering the recent global financial crisis. In a nutshell, these securities are portfolios consisting of a large number of individual residential mortgage loans. It is well known that individual mortgage loans can be prematurely terminated because a borrower has either prepaid or defaulted on the loan. Rather shockingly, the models used by rating agencies, in particular, seemed to largely understate the possibility of borrower default. The aim of this paper is to present an option-based framework for pricing MBSs. It incorporates both defaults and prepayments of individual mortgages. It compares two different approaches for modelling mortgage termination and studies the importance of default in the model.

Historically there have been two different approaches to modelling MBSs – econometric and option-based models.

Econometric models focus on MBS asset as a whole. They specify a particular functional relationship that captures mortgage termination patterns. They are, then, calibrated on historical termination data and used to project cash flows over the life of the security. Different explanatory variables can be used, such as: ratio of coupon and long-term interest rate (to capture an incentive to refinance), pool factor (to capture burnout effect), a seasonal dummy variable (to capture seasoning effect), macroeconomic variables such as level, slope, and recent history of interest rates, change in housing prices, unemployment rate, GDP growth, or different pool characteristics that capture borrowers' personal characteristics<sup>1</sup>. These models typically apply a forward-looking pricing technique based on Monte Carlo simulation.

Option-based models start with individual mortgages. These models treat the mortgage as a bond with two embedded options: an American call option (the option to prepay) and a series of European put options (the option to default). There are several models of pricing individual mortgage loans. These models vary in the manner in which they address the borrower's decision to terminate the mortgage and in the manner in which they treat the heterogeneity of borrowers. As a borrower's termination decision is based on his/her anticipation of the future,

---

<sup>1</sup> Some types of functional forms and different choices of explanatory variables can be found in Schwartz and Torous (1989), Richard and Roll (1989) and Spahr and Sunderman (1992).

a backward technique is typically employed for solving the valuation problem of an individual mortgage loan.

The recent financial crisis pointed out the weak spots in the financial system as a whole. Among them, it became obvious that the models used by financial institutions failed to adequately price risks related to MBSs. Careful modelling of default rose to prominence.

Historically financial institutions primarily used econometric models. These models did not put much emphasis on default as mortgage pools were mostly seen as of low default risk due to agencies' guarantees, while securitization was perceived to mitigate default risk. Thus the issue of default was either completely ignored or treated exogenously by projecting a certain default rate<sup>2</sup>. This was partly due to insufficient data on individual contracts underlying the security, aggregated information, and short history of available data. Schwartz and Torous (1992) stress the importance of joint consideration of prepayment and default, acknowledging that defaults can significantly change patterns of MBS cash flows.

Until the crisis hit, option-based MBS pricing models were mostly ignored by practitioners. Apart from being more complicated to implement than their econometric counterparts, they were criticized for imposing an upper bound to the values of MBSs that is not observed in practice<sup>3</sup>.

Early option-based models addressed the borrower's decision assuming interest rate as the only source of risk. For example, Dunn and McConnell (1981) modelled MBS as an annuity with embedded American call option, ignoring default and assuming no transaction costs related to prepayments<sup>4</sup>. These models focused more on the economic environment and backward algorithm applied than on careful modelling of a borrower's decision.

---

2 As late as 2001, historical evidence pointed to low rates of default (less than 0.5% of conditional prepayment rate on average). Based on this historical data, many investment banks modelled default using the Standard Default Assumption Curve developed by the Bond Market Association. See Salomon Smith Barney (2001).

3 The MBS prices obtained were capped by par plus transaction costs, see Stanton (1995) and Longstaff (2002).

4 Discussion of the early models in literature can be found in Hendershott and VanOrder (1987). See also Kau et al. (1992), Kau and Keenan (1995), and Kalotay, Yang and Fabozzi (2004).

Different authors note that borrowers often do not prepay optimally. In order to capture that fact they introduce transaction costs and other sorts of frictions<sup>5</sup>. However, all these early models assume homogeneity of classes of borrowers, so that refinancing occurs simultaneously in the entire pool. Thus, when interest rates reach the critical level again, there will be no further borrowers to prepay. These models lack power to explain seasonality and to capture the possibility that some borrowers prepay when it is financially not optimal to do so, or forego prepaying when it is financially optimal.

In order to overcome this problem, Stanton (1995) introduces heterogeneity of borrowers. He assumes that mortgagors face heterogeneous transaction costs measured as fractions of the unpaid mortgage balance and governed by beta distribution. Further, he assumes a probability of exogenous prepayment modelled by a hazard function. The assumption that a prepayment decision can be made only at random discrete time intervals prevents the situation from occurring in which all borrowers of a certain transaction cost level prepay immediately once the interest rate reaches a critical level. While this was a significant modelling breakthrough, there was one major drawback. Namely, the interest rate is assumed to be the sole source of risk. Stanton's work is later extended by Downing, Stanton and Wallace (2003) to include the possibility of default.

From the beginning of the option-based approach to mortgage valuation, default was recognized as part of a borrower's decision<sup>6</sup>. Various authors recognize the default option as an inseparable part of valuation, which as such should be simultaneously modelled with prepayment<sup>7</sup>. The source of uncertainty that influences a borrower's decision to default is the value of the collateral, i.e. the level of house prices. Kau and Slawson (2002) introduce frictions in this framework to account for heterogeneity of borrowers and for exogenous terminations resulting from the borrower's characteristics<sup>8</sup>.

Prepayment and default options compete with each other in that by exercising one the other is rendered worthless. Namely, if a borrower defaults s/he cannot prepay and vice-versa. The borrower, in a rational framework, chooses the timing

---

5 Early models already recognized that borrowers often do not act in a rational manner. For example, Dunn and McConnel (1981) allowed for sub-optimal prepayments triggered by realizations of a Poisson process. For more references, see Kalotay, Yang and Fabozzi (2004).

6 See Bardhan et al (2006), Kau et al. (1992) and Kau and Keenan (1995).

7 See Kau et al. (1992), Kau and Keenan (1995), Deng, Quigley and Van Order (1995).

8 The idea of heterogeneity of borrowers is also found in Deng, Quigley and Van Order (2000).

and type of mortgage termination that minimizes his/her liability. Thus, these two options must be valued simultaneously.

More recently, Kalotay, Young and Fabozzi (2004) introduce the idea of using two different yield curves when pricing MBS – one for modelling the refinancing strategy of borrowers, the other for discounting MBS cash flows. Longstaff (2002) incorporated the credit spread of the borrower into his refinancing decision. He also applied Longstaff and Schwartz least-squares method to solve for optimal strategy. A group of authors (Goncharov (2005), Kolbe and Zagst (2007)) emphasize the intensity-based approach to value mortgage contract and MBS.

In this paper we focus on fixed-rate residential pass-through MBSs as the most common segment of the market. We present two models to value such securities. In particular, we combine heterogeneity of borrowers as modelled in Downing, Stanton and Wallace (2003) with a specific option-based model for pricing individual mortgages. One is the model presented in Downing, Stanton and Wallace (2003) while the other is the mortgage valuation technique of Kau and Slawson (2002). Our paper is, to the best of our knowledge, the first paper to do so. We compare and contrast the two methods of pricing pass-through securities focusing, in particular, on the role of borrowers' defaults. Both of these models are flexible and can capture many realistic features of borrower behaviour.

## **2. VALUATION PROCEDURE**

The modelling takes a bottom-up approach and views MBS as a three level asset – underlying variables, mortgage, and MBS. The valuation framework is set up with respect to each of the three levels. A bivariate binomial option pricing technique is applied to solve the mortgage contract partial differential equation.

We have chosen two approaches to model the borrower's decision to terminate, one, which we refer to as the KS model, in the framework of Kau and Slawson (2002), and the other, which we refer to as the DSW model, in the framework of Downing, Stanton and Wallace (2003).

Furthermore, obtaining the MBS value is achieved by summing up the values of individual contracts, taking into account heterogeneity of borrowers in the same manner as in Stanton (1995) and DSW.

## 2.1 Underlying variables

The economic environment is defined by two risk-neutral stochastic processes<sup>9</sup>.

The term structure of interest rates is modelled as a one-factor Cox, Ingersoll and Ross (1985) (CIR) model with the long-run interest rate  $\theta$ , the speed of adjustment towards the long-run rate  $k$ , interest rate volatility  $\sigma_r$ , and a source of uncertainty given by the standardized Wiener process  $Z_r$ . The model is chosen due to its ability to incorporate the mean reversion, non-constant volatility, non-negative values of interest rates and the existence of the closed-form solution for the price of a zero coupon bond. Furthermore, it is assumed that the interest rate risk premium is absorbed by parameters  $k$  and  $\theta$  (see the first expression in (1)).<sup>10</sup>

The house price process, with respect to a risk neutral measure  $Z_H$ , the standardized Wiener process, is modelled by a geometric Brownian motion process with the drift  $(r-s)$  and house price volatility  $\sigma_H$ . Here, quantity  $s$  is service flow provided by the house. The two Brownian motion processes are correlated and the coefficient of correlation is equal to  $\rho$ .

The space of underlying variables is thus given by the following system of equations:

$$\begin{cases} dr = k(\theta - r)dt + \sigma_r \sqrt{r} dZ_r, r(0) = r_0 \\ dH_t = H(r - s)dt + H\sigma_H dZ_H, H(0) = H_0 \\ dZ_r dZ_H = \rho dt \end{cases} \quad (1)$$

## 2.2 Bivariate binomial pricing technique

Bivariate binomial pricing technique<sup>11</sup> is used for pricing contingent claims that depend on two state variables. This is done by constructing a bivariate binomial tree. The process consists of the following steps.

<sup>9</sup> See Hilliard, Kau and Slawson (1998).

<sup>10</sup> This is in line with the Local Expectations Hypothesis. For details, see Kau et al. (1992: 281) and Kau and Keenan (1995: 222). For modelling of term structure of interest rates in Serbia see, for example, Drenovak and Urošević (2010).

<sup>11</sup> A detailed description of this procedure with all derivations can be found in Hillard, Kau and Slawson (1998) and Hillard and Schwartz (1996).

The first step is transformation of the underlying variables given by (1) through two consequent changes of variables into two non-correlated processes with constant volatilities. The first change of variables, given by:

$$\begin{cases} S = \ln H \\ R = 2\sqrt{r} \end{cases} \quad (2)$$

transforms the underlying variables given by (1) into the processes with constant volatilities. The second change of variables, given by:

$$\begin{cases} X_1 = \sigma_r S + \sigma_H R \\ X_2 = \sigma_r S - \sigma_H R \end{cases} \quad (3)$$

transforms the process (S,R) into non-correlated processes, preserving constant volatility. Furthermore, processes  $dX_i$ ,  $i=1,2$ , can be represented in the form:

$$dX_i = \mu_i dt + \sigma_i dZ_i, \quad i = 1,2 \quad (4)$$

where processes  $Z_i$ ,  $i=1,2$ , are chosen such that

$$\begin{cases} dZ_H + dZ_r = \sqrt{2(1+\rho)}dZ_1 \\ dZ_H - dZ_r = \sqrt{2(1-\rho)}dZ_2 \end{cases} \quad (5)$$

Constant volatilities allow construction of a recombining tree, while the absence of correlation allows multiplying marginal transition probabilities in order to obtain joint probability of reaching any particular node.

Next, a bivariate binomial tree is constructed. From each node in the tree, four possible nodes can be reached in the subsequent time step. These nodes correspond to combinations of the up and down movements of the two transformed underlying variables. The choice of jump sizes and corresponding transition probabilities is made so as to guaranty weak convergence of the constructed tree to the transformed two-dimensional process<sup>12</sup>. This is achieved by allowing multiple jumps, such that  $X_i + \mu_i \Delta t$  is situated between the nodes reachable in the next time step. For each of the transformed underlying variables  $X_1$  and  $X_2$ , the

---

<sup>12</sup> As a compromise between establishing convergence and saving execution time we use three time steps per month in our simulations, based on the convergence check performed on a hypothetical one-year mortgage by increasing the number of time steps per month.

increment up-step size and the corresponding probability for the next time step are chosen as:

$$\begin{cases} (2k_i + 1)\sigma_i\sqrt{dt}, \text{ up - movement step size increment} \\ \frac{1}{2} - k_i + \frac{\mu_i\sqrt{dt}}{2\sigma_i}, \text{ up - movement probability} \end{cases} \quad (6)$$

The values of the underlying variables  $r$  and  $H$  are, then, computed for each node using inverse transformations. The algorithm works backwards through the tree, calculating the value of the contingent claim at each time step based upon values of the underlying variables and derivative values already calculated for the future time steps.

### 2.3 Valuation of individual mortgage contracts

The price of the mortgage contract, as in Kau and Keenan (1995: 225) follows the second-order partial differential equation (PDE):

$$\begin{aligned} & \frac{\partial f}{\partial t} + (r - s)\frac{\partial f}{\partial H}H + k(\theta - r)\frac{\partial f}{\partial r} \\ & + \frac{1}{2}\left(\sigma_r^2 r \frac{\partial^2 f}{\partial r^2} + \sigma_H^2 H^2 \frac{\partial^2 f}{\partial H^2} + 2\rho\sigma_r\sigma_H\sqrt{r}H \frac{\partial^2 f}{\partial r\partial H}\right) = rf \end{aligned} \quad (7)$$

subject to the following boundary conditions:

$$\begin{cases} f(t, r, 0) = 0, \text{ as zero house price induces borrower to default} \\ \lim_{r \rightarrow \infty} f(t, r, H) = 0, \text{ as zero disc. factor implies zero value of future payments} \\ f(T, r, H) = 0, \text{ as mortgage value is zero at termination} \end{cases} \quad (8)$$

In the rational framework, the borrower minimizes his/her mortgage liability, given three actions – continuation of monthly payments, prepayment, or default<sup>13</sup>. Which of the three actions she chooses depends on which of them leads to the smallest mortgage value (obligation).

Prepayment is possible at any moment over the life of a mortgage, while default can be rational only at payment dates as the borrower can benefit from living in the house up to the payment date. As prepayment gives the borrower the right

<sup>13</sup> We assume that no partial prepayment is possible.

to pay off his/her mortgage and receive the house, it is modelled as an American call option on the present value of the remaining payments with a mortgage balance as a strike price. On the other hand, the default option is a right to cease payments and abandon ownership of the house. Thus it is modelled as a series of linked European put options with the house price as the underlying asset and the present value of the remaining payments as a strike price. Since by exercising one option the borrower loses the other, these two options have to be considered (and valued) jointly.

We furthermore introduce two different approaches to the mortgage termination decision of an individual borrower. The KS model treats termination endogenously and considers notions of different frictions. In the DSW model, termination is treated exogenously and is governed by a hazard rate that depends on the regions where it is optimal to exercise each of the options.

The models can be solved going backwards through the grid created by discretization of the transformed processes. At each node the values of corresponding underlying variables are restored through inverse transformations and the values of prepayment and default options are determined simultaneously, upon which the value of the mortgage contract is obtained.

### **KS model**

We first present the mortgage valuation methodology as in the KS model. The investor in the mortgage is long in the series of payoffs coming from the mortgage and short in options given to the borrower. Thus at each moment in time the value of the mortgage  $V(t,r,H)$  is equal to the present value of the remaining payments (PVRP) minus the value of embedded options

$$V(t,r,H) = PVRP(t,r) - P(t,r,H) - D(t,r,H), \quad 0 \leq t \leq T \quad (9)$$

Here  $P(t,r,H)$  is the value of the option to prepay and  $D(t,r,H)$  is the value of the option to default at time  $t$ , while  $T$  is time to maturity of the mortgage contract.

The borrower's decision, in the rational framework, is based upon choosing the action that leads to the smallest value of his/her liability. The values of prepayment and default options (value of one option in the presence of the other) are given by the following set of equations:

$$\begin{aligned}
 P(t,r,H) &= \begin{cases} PVRP(t,r) - B(t), & \text{prepayment} \\ 0, & \text{default} \\ PVP(t,r,H), & \text{continuation} \end{cases} \\
 D(t,r,H) &= \begin{cases} 0, & \text{prepayment} \\ PVRP(t,r) - H, & \text{default} \\ PVD(t,r,H), & \text{continuation} \end{cases}
 \end{aligned} \tag{10}$$

where  $B(t)$  is the mortgage balance at time  $t$ ,  $PVP(t,r,H)$  is the present value of future prepayment options at time  $t$  given  $r$  and  $H$  and  $PVD(t,r,H)$  is the present value of future default options at time  $t$  given  $r$  and  $H$ . The chosen scenario at time  $t$  is the one that leads to the smallest value of the mortgage:

$$V(t,r,H) = \begin{cases} \min\{MP, H\}, & \text{just before termination} \\ \min\{B, H, PVRP - PVP - PVD\}, & \text{just before any other payment} \\ \min\{B, PVRP - PVP - PVD\}, & \text{any other moment} \end{cases} \tag{11}$$

where  $MP$  is a mortgage payment.

The model is further extended to include frictions of three different types: transaction costs, suboptimal termination, and decision probability.

The transaction costs facing a borrower are motivated by different reactions of borrowers to the changes in the underlying variables. They are introduced to explain the different termination decisions of borrowers faced with the same economic incentives, due to the different characteristics of borrowers. These costs refer to various costs associated with prepayment or default such as costs of making a prepayment decision, cost of moving, etc.

Transaction costs are modelled to differentiate between borrowers by changing the values of parameters upon which the borrower's decision is made (in our case, house price and mortgage balance), in the following manner:

$$\begin{cases} B(t)^{opt} = B(t)(1 + \tau_{var}^{opt}) + B(0)\tau_{fix}^{opt}, & \text{for the mortgage balance} \\ H(t)^{opt} = H(t)(1 + v_{var}^{opt}) + H(0)v_{fix}^{opt}, & \text{for the house price} \end{cases} \tag{12}$$

where  $\tau$  is a respective transaction cost of a certain borrower associated with prepayment and  $v$  associated with default and  $B(t)$  is the mortgage balance at

time  $t$ . Transaction costs are allowed to be fixed (measured as a percentage of the initial parameter value) or variable (measured as a percentage of the current parameter value).

As transaction costs are paid by the borrower, they affect the value of the mortgage liability facing the borrower. On the other hand, as they alter cash flows coming from the asset by influencing the borrower's decision, they also affect the value of the mortgage asset to the investor in an indirect manner. The different impact of transaction costs on these two values results in different values of mortgage liability to the borrower and mortgage asset to the investor.

Suboptimal termination refers to termination of a mortgage due to non-financial exogenous reasons, such as a change in the borrower's personal circumstances. Being exogenous to the model this decision does not depend on the movements of the underlying variables, and as such is incorporated into the model by introducing the expected rate of suboptimal termination  $\lambda(t)$ , which is usually projected in line with the PSA schedule, to imitate low termination rates in the initial months. However, the choice of type of termination (i.e. termination through prepayment or through default) depends on the underlying variables.

Furthermore, transaction costs of suboptimal termination associated with default and prepayment are added to differentiate between the borrowers. These costs are assumed to be lower than transaction costs of optimal termination<sup>14</sup>. It is also assumed that suboptimal termination can occur only at payment dates, for simplicity reasons.

Decision probability accounts for the possibility of non-termination of the mortgage when it is financially optimal to do so. It is also treated exogenously and is modelled as the probability of a decision on termination being made at a particular point in time<sup>15</sup>.

In the presence of frictions, the value of the mortgage liability/asset is the weighted sum of possible choices made by the borrower and is given by:

---

<sup>14</sup> The reason for this is that borrower is already facing a termination of mortgage. S/he only needs to differentiate between the types of termination and thus is not incurring the same costs as in the case of optimal termination. For modelling purposes they are assumed to be half of the value of the respective transaction costs of optimal termination. Additionally, we assume throughout our simulations that transaction costs related to prepayment and default are equal to each other.

<sup>15</sup> For modelling purposes it is assumed to be 0.5.

$$\begin{aligned}
 V^L(t, r, H) &= \lambda(t)V_{sub}^L(t, r, H) + (1 - \lambda(t))(\delta V_{opt}^L(t, r, H) + (1 - \delta)V_{nt}^L(t, r, H)), \\
 V^A(t, r, H) &= \lambda(t)V_{sub}^A(t, r, H) + (1 - \lambda(t))(\delta V_{opt}^A(t, r, H) + (1 - \delta)V_{nt}^A(t, r, H)),
 \end{aligned} \tag{13}$$

where subscripts *opt*, *sub*, and *nt* stand for the respective values in the cases of optimal termination, suboptimal termination, and non-termination. The values  $V_{opt}^L/V_{opt}^A$  and  $V_{sub}^L/V_{sub}^A$  are induced by the movements of the underlying variables and the corresponding transaction costs and are thus determined endogenously.

The value of mortgage liability in case of optimal termination is calculated as the minimum of the values when prepayment, default, or continuation is chosen:

$$V_{opt}^L(t, r, H) = \min\{B(t)^{opt}, H(t)^{opt}, PVRP - PVP^A - PVD^A\}, \tag{14}$$

where  $PVP^A/PVP^L$  and  $PVD^A/PVD^L$  are the present values of the prepayment and default options assets/liabilities.

Last but not least, the value of mortgage liability in the case of suboptimal termination is calculated as a minimum of the values when suboptimal termination occurred due to prepayment or default.

$$V_{sub}^L(t, r, H) = \min\{B(t)^{sub}, H(t)^{sub}\} \tag{15}$$

### DSW model

The DSW model applies a different pattern for mortgage termination. It uses the following hazard rate to trigger mortgage termination:

$$\begin{aligned}
 \lambda(t) &= \lambda_0(t) + \beta_2 D_t + \beta_3 LTV_t P_t, \\
 \lambda_0(t) &= \beta_0 \arctg\left(\frac{t}{\beta_1}\right)
 \end{aligned} \tag{16}$$

where  $D_t$  is an indicator when default or prepayment is optimal,  $P_t$  is an indicator when prepayment is optimal,  $\lambda_0(t)$  is a background hazard rate, and LTV stands for loan-to-value ratio.

The background hazard function accounts for mortgage terminations triggered by exogenous events and is not related to the movements of the underlying variables. It is chosen in the spirit of Richard and Roll (1989), motivated by the seasoning

of mortgages. The choice of parameters  $\beta_1$  and  $\beta_0$  determines the maximum rate achieved and the speed of approach to that maximum.

The second component of the hazard rate is dependant on the region where optimal termination is chosen, and thus connects the hazard rate to endogenous incentives. The magnitude of the effect of the movements of the underlying variables is controlled by the choice of parameter  $\beta_2$ .

The third component is motivated by the reliance of prepayments on the credit constraints or underwriting requirements of the borrower, which is captured by the LTV ratio.

The value of the mortgage liability is then given by the weighted sum of values in the case of the decision not to terminate and the decision to terminate:

$$\begin{aligned} V^L &= (1-p)V_{cont}^L + pB(1+X)I_1 + pH(1+X)I_2 \\ p &= 1 - e^{-\frac{\lambda}{12}} \end{aligned} \tag{17}$$

Here,  $V_{cont}^L$  is the value of the mortgage liability when none of the options is exercised,  $X$  is the transaction costs level (assumed equal for prepayment and default),  $I_1/I_2$  are the indicators of prepayment/default occurrence and  $p$  is the probability of termination given the hazard rate (16). In the case of the decision to terminate, the type of termination is determined optimally.

We note that the regions where it is optimal to exercise each of the options or to continue with the mortgage coincide in the two models. Thus the only true difference between the two described models is in the decision process, summarized in formulas (13) and (17). How different the MBS values obtained by these two models will be depends on the choice of parameters for the hazard rate inducing mortgage termination (given by (16)) in the case of the DSW model, and the expected rate of suboptimal termination  $\lambda(t)$  and decision probability  $\delta$  in the case of the KS model.

## **2.4 MBS valuation**

The MBS value is obtained as a weighted sum of the different types of mortgages in the pool that are obtained for various levels of transaction costs. We assume beta distribution for transaction costs in line with Stanton (1995). Beta distribution is

chosen because of the diversity of different shapes it can take for different choices of parameter values and for being defined on the bounded interval.<sup>16</sup>

We assume a number N of different classes of borrowers represented by a certain level of transaction costs. Different levels of transaction costs are specified by discretizing of beta distribution by taking its quantiles<sup>17</sup>.

### 3. THE NUMERICAL RESULTS

This section compares two pricing models. In particular it studies numerically sensitivity of the pricing results, with respect to changes in model parameters. The parameters related to economic environment, contract, and parameters describing heterogeneity of borrowers, are chosen in line with Kau and Slawson (2002) and Downing, Stanton and Wallace (2003).

#### 3.1. Sensitivities with respect to changes in parameter values

##### *Sensitivities towards changes in beta distribution parameters*

Beta distribution determines the choice of the transaction cost level for different classes of borrowers. The steepness of the corresponding cumulative distribution function, which is governed by the choice of parameters of the beta distribution, defines if lower or higher transaction costs will be favoured.

Table 1 represents sensitivities of the MBS prices towards the parameters of the beta distribution<sup>18</sup>. It can be observed that an increase in parameter  $\alpha$  leads to an increase in the MBS value, while an increase in  $\beta$  has the inverse effect.

---

16 The probability density function  $f(x; \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 y^{\alpha-1}(1-y)^{\beta-1} dy}$ ,  $x \in (0,1)$  of the beta distribution

is defined on the bounded set, which sets an upper bound for the transaction cost.

17 This is achieved by dividing the set [0,1] into N equal segments, and assuming the central point of each segment as representative for the corresponding class of borrowers. The respective transaction cost levels for each class of borrowers are obtained by applying the inverse of the cumulative distribution function to the central points.

18 The mean transaction cost is then given by  $\alpha(\alpha+\beta)^{-1}$ , while variance is given by  $\alpha\beta(\alpha+\beta)^{-2}(\alpha+\beta+1)^{-1}$ . Note that we have chosen beta distribution (in line with DSW) such that  $\alpha < 1$  and  $\beta \geq 1$ , for which its probability density function is strictly decreasing, implying a higher probability for the lower transaction costs. For comparison reasons, we use the parameters of the beta distribution as in DSW, namely,  $\alpha=0.3779162$ ,  $\beta=2.8838$ .

An increase in  $\beta$  lowers the expectation and dispersion, resulting in a higher probability of lower transaction costs, which decreases the value of MBS. An increase in  $\alpha$  increases both the expectation and dispersion and results in slightly higher transaction costs and thus higher MBS values.

**Table 1. MBS value and variation of the beta distribution parameters**

KS model					DSW model				
$\alpha / \beta$	1	2	3	4	$\alpha / \beta$	1	2	3	4
0.1	99.33	99.11	98.92	98.88	0.1	99.38	99.26	99.12	99.08
0.2	99.29	99.28	99.31	99.25	0.2	99.33	99.32	99.33	99.32
0.4	99.70	99.61	99.39	99.30	0.4	99.56	99.55	99.42	99.35
0.6	99.79	99.68	99.66	99.58	0.6	99.66	99.56	99.56	99.51

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, initial spot rate 10%, steady-state spot rate 10%, interest rate st.dev. 10%, speed of adj. 0.25, house price volatility 10%, loan-to-value ratio 90%, assuming no correlation between the two Brownian motion processes and three levels of transaction costs.  $\lambda$  parameters for DSW model:  $\beta_0=0.1$ ,  $\beta_1=1.5$ ,  $\beta_2=0.4$ ,  $\beta_3=1.1$ .

*Sensitivity towards parameters inducing mortgage termination*

As mentioned above, termination in the KS model is induced by the probability of suboptimal termination which is projected in line with the PSA schedule.<sup>19</sup>

In Table 2 we observe that an increase in probability of suboptimal termination (an increase in PSA) lowers the value of MBS in the upward sloping yield curve environment, while when the yield curve is flat or downward sloping, the MBS value increases. This effect can be explained by more cash flows being returned to the investor due to suboptimal termination that, in the case of low interest rate levels, can be reinvested on less favourable terms (and vice-versa).

---

<sup>19</sup> On PSA schedules, a de-facto industry standard on the MBS market, see e.g. Fabozzi (2004).

**Table 2. MBS and mortgage asset values for different transaction costs in the KS model and variation of  $\lambda$  parameter**

PSA	$r_0$	MBS	MBS1	MBS2	MBS3	PSA	$r_0$	MBS	MBS1	MBS2	MBS3
0	6%	103.15	101.27	101.33	106.84	2	6%	103.05	101.27	101.34	106.55
	10%	99.35	98.89	99.03	100.14		10%	99.56	99.11	99.30	100.28
	14%	93.72	93.57	93.67	93.94		14%	94.28	94.13	94.23	94.46
0.1	6%	103.14	101.27	101.34	106.83	5	6%	102.91	101.27	101.31	106.14
	10%	99.36	98.90	99.04	100.14		10%	99.83	99.41	99.64	100.45
	14%	93.75	93.60	93.70	93.97		14%	94.99	94.87	94.96	95.13
0.5	6%	103.13	101.27	101.34	106.77	8	6%	102.78	101.27	101.31	105.76
	10%	99.41	98.94	99.10	100.17		10%	100.06	99.69	99.93	100.57
	14%	93.87	93.71	93.82	94.08		14%	95.58	95.47	95.57	95.68
1	6%	103.10	101.27	101.34	106.70	10	6%	102.70	101.27	101.30	105.53
	10%	99.46	99.00	99.17	100.21		10%	100.19	99.85	100.09	100.63
	14%	94.01	93.86	93.96	94.21		14%	95.91	95.81	95.92	96.00

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, steady-state spot rate 10%, interest rate volatility 10%, speed of adj. 0.25, house price volatility 10%, loan-to-value ratio 90%, and no correlation. MBS<sub>i</sub>, i=1,2,3, are mortgage asset values for low, medium and high transaction costs.

**Table 3. MBS and mortgage assets values for different transaction costs in the DSW model and variation in hazard rate parameters**

$\beta_0$	$\beta_2$	$\beta_3$	MBS	MBS1	MBS2	MBS3	$\beta_0$	$\beta_2$	$\beta_3$	MBS	MBS1	MBS2	MBS3
0	0	-2	116.09	132.38	115.76	100.14	1	0	-2	109.90	123.38	108.35	97.97
		-1	101.79	103.22	102.01	100.14			-1	99.88	102.41	99.25	97.97
		0	100.14	100.14	100.14	100.14			0	98.65	100.08	97.90	97.97
	0.5	1	99.76	99.51	99.62	100.14		1	97.63	97.39	97.53	97.97	
		2	99.62	99.30	99.42	100.14		2	97.54	97.27	97.38	97.97	
		-2	103.42	106.43	103.68	100.14		0.5	-2	101.04	104.72	100.42	97.97
-1	100.35	100.48	100.45	100.14	-1	98.83	100.38		98.13	97.97			
0	99.82	99.60	99.72	100.14	0	97.67	97.45		97.60	97.97			
1	0.5	1	99.66	99.38	99.47	100.14	1	97.57	97.30	97.42	97.97		
		2	99.54	99.17	99.33	100.14	2	97.47	97.13	97.31	97.97		
		-2	100.77	101.21	100.96	100.14	1	-2	99.12	100.89	98.50	97.97	
	-1	99.92	99.77	99.86	100.14	-1		98.49	99.81	97.70	97.97		
	0	99.69	99.40	99.52	100.14	0		97.58	97.31	97.46	97.97		
	1	1	99.56	99.20	99.36	100.14	1	97.49	97.16	97.34	97.97		
2		99.51	99.12	99.27	100.14	2	97.44	97.09	97.27	97.97			

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, initial spot rate 10%, steady-state spot rate 10%, interest rate volatility 10%, speed of adj. 0.25, house price volatility 10%, loan-to-value ratio 90%, and no correlation. Parameters  $\lambda$  for the DSW model are selected as follows:  $\beta_1=1$ . MBS<sub>i</sub>, i=1,2,3, are mortgage asset values for low, medium and high transaction costs.

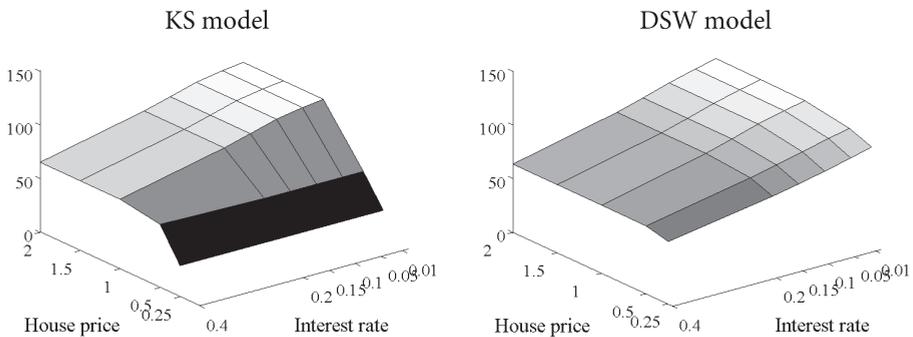
In the case of the DSW model, we observe that increased probability of suboptimal termination, either through the presence of background hazard rate or through increase in the values of parameters  $\beta_2$  and  $\beta_3$ , lowers the value of MBS to the investor in the chosen economic environment. The same pattern can be observed in the case of mortgage assets for different levels of transaction costs.

We note that mortgage asset values for borrowers with high transaction costs remain unchanged if one varies parameters  $\beta_2$  and  $\beta_3$ . This can be explained by high transaction costs that push both options out of money. As a result, mortgage termination is determined solely by the background hazard rate.

*Sensitivity towards initial values of house price and interest rates*

MBS value is affected in the same manner as the individual mortgage contract when it comes to the initial values of the underlying variables, as it is obtained by averaging the values of the single mortgage assets for different classes of borrowers. This can be seen by comparing the shape of MBS value with respect to different parameter values (Figure 1) and the shape of the mortgage liability value presented for borrowers of the lowest level of transaction costs (Figure 2).

**Figure 1. Variation in the values of  $r_0$  and  $H_0$  of MBS value**



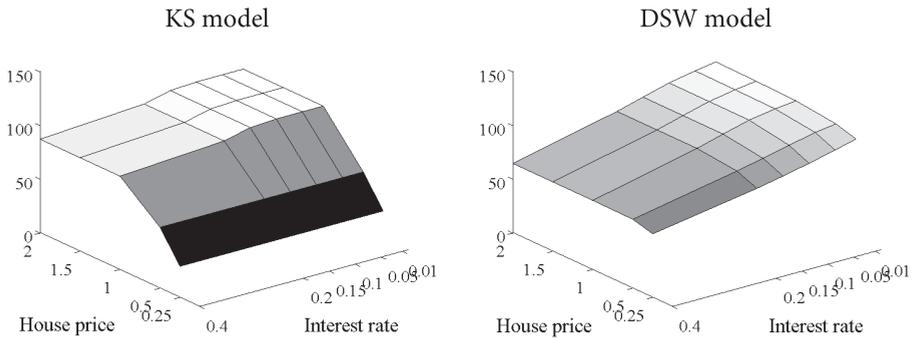
Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, steady-state spot rate 10%, speed of adj. 0.25, interest rate st.dev. 10%, house price st.dev. 10%, loan-to-value ratio 90%, no correlation, three levels of transaction costs.  $\lambda$  parameters for DSW model:  $\beta_0=0.1$ ,  $\beta_1=1.5$ ,  $\beta_2=0.4$ ,  $\beta_3=1.1$ .

We observe that the rate at which MBS value changes with respect to different choices on initial parameter values differs in the two models, due to different triggers of mortgage termination. It can be observed that the change in underlying

variables has a higher impact on the endogenously induced termination, especially observed in the case of low initial values of house price.

The obtained shape is primarily governed by the interaction of the different components of the single mortgage contract. The mortgage value is driven by the combined effect of house price and interest rate levels on the present value of the remaining payments (PVRP) and the value of the two options. Low house price levels increase the option to default and thus decrease the mortgage value. With an increase in house price levels the prepayment option becomes more important. On the other hand interest rate level is the main driver of the values of both prepayment option and the present value of the remaining payments. These two quantities have an opposite impact on mortgage value. However as the prepayment option value is significantly lower than the PVRP, the interest rate effect on the mortgage value is mainly driven by the effect of interest rates on the PVRP.

**Figure 2. Variation in the values of  $r_0$  and  $H_0$  of mortgage liability for borrower with the lowest transaction cost**



*Sensitivities towards loan-to-value ratio and interest rate and house price volatilities*

As MBS value is obtained as the average of the values of mortgage assets for different transaction costs level, its sensitivity towards a parameter choice is derived from the sensitivity of the mortgage value. To give more insight as to how this value is generated we also include the values of mortgage liabilities in cases of different classes of borrowers. We observe that for each of the two models the value of mortgage liability rises with an increase in transaction costs. Higher transaction costs reduce the region where it is optimal to exercise the respective option and thus lower its value, which in turn raises the mortgage value.

**Table 4. Variation in loan-to-value ratio and volatilities of the MBS and mortgage liability values for different transaction costs**

LTV	$\sigma_r$	$\sigma_H$	KS model				DSW model			
			MBS	MBS1	MBS2	MBS3	MBS	MBS1	MBS2	MBS3
0.8	0.05	0.05	99.9805	99.8038	101.2429	102.0546	99.6751	99.8821	100.8386	100.8479
		0.1	99.9802	99.8031	101.2428	102.0546	99.6746	99.8807	100.8385	100.8479
		0.2	99.7207	99.5685	101.0081	102.0161	99.6529	99.8591	100.8154	100.8455
	0.1	0.05	99.4745	98.9975	100.9956	102.2151	99.4141	99.3420	100.7790	101.0231
		0.1	99.4737	98.9957	100.9950	102.2151	99.4134	99.3390	100.7781	101.0231
		0.2	99.2578	98.7944	100.7686	102.1686	99.3921	99.3186	100.7548	101.0199
	0.2	0.05	98.7385	97.5885	100.0712	102.8222	99.0578	98.4805	100.3808	101.6847
		0.1	98.7344	97.5808	100.0667	102.8224	99.0504	98.4750	100.3767	101.6847
		0.2	98.5306	97.4189	99.8663	102.7440	99.0215	98.4535	100.3539	101.6782
0.9	0.05	0.05	99.9805	99.8038	101.2429	102.0546	99.6727	99.8573	100.8381	100.8479
		0.1	99.9600	99.7844	101.2336	102.0542	99.6716	99.8552	100.8376	100.8479
		0.2	98.9733	98.7067	100.3173	101.8905	99.5729	99.7631	100.7383	100.8361
	0.1	0.05	99.4745	98.9975	100.9956	102.2151	99.4042	99.3019	100.7694	101.0231
		0.1	99.4602	98.9812	100.9837	102.2148	99.4025	99.2988	100.7679	101.0231
		0.2	98.5603	98.0242	100.0826	102.0247	99.3028	99.2213	100.6726	101.0083
	0.2	0.05	98.7385	97.5885	100.0712	102.8222	99.0030	98.4138	100.3395	101.6847
		0.1	98.7232	97.5695	100.0538	102.8216	98.9981	98.4061	100.3353	101.6847
		0.2	97.8536	96.7632	99.2192	102.5329	98.9116	98.3372	100.2572	101.6580
1	0.05	0.05	99.6939	99.2968	101.2132	102.0533	99.6581	99.8196	100.8367	100.8479
		0.1	99.1102	98.5141	100.7276	102.0436	99.6159	99.7678	100.8053	100.8478
		0.2	96.8678	96.1167	98.3578	101.5045	99.3085	99.4814	100.4952	100.8025
	0.1	0.05	99.2585	98.6789	100.9493	102.2138	99.3794	99.2574	100.7583	101.0231
		0.1	98.7280	97.9614	100.4492	102.2025	99.3348	99.2151	100.7240	101.0230
		0.2	96.5601	95.6410	98.1429	101.5995	99.0519	98.9557	100.4301	100.9691
	0.2	0.05	98.5816	97.3958	100.0108	102.8209	98.9528	98.3429	100.3005	101.6847
		0.1	98.1057	96.8177	99.5157	102.7962	98.9225	98.3071	100.2699	101.6838
		0.2	95.9026	94.6806	97.3770	101.9666	98.6745	98.0868	100.0265	101.5980

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, initial spot rate 10%, steady-state spot rate 10%, speed of adjustment 0.25 and no correlation between the two sources of uncertainty.  $\lambda$  parameters for DSW model:  $\beta_0=0.1$ ,  $\beta_1=1.5$ ,  $\beta_2=0.4$ ,  $\beta_3=1.1$ . MBS<sub>i</sub>, i=1,2,3 are mortgage liabilities values for low, medium and high transaction costs.

Higher volatility increases the probability of reaching the exercise region of the respective option and thus increases the value of the option and decreases the value of the mortgage. However this impact is offset by the competing effect of the other option. Furthermore the main driver of the mortgage value and of the

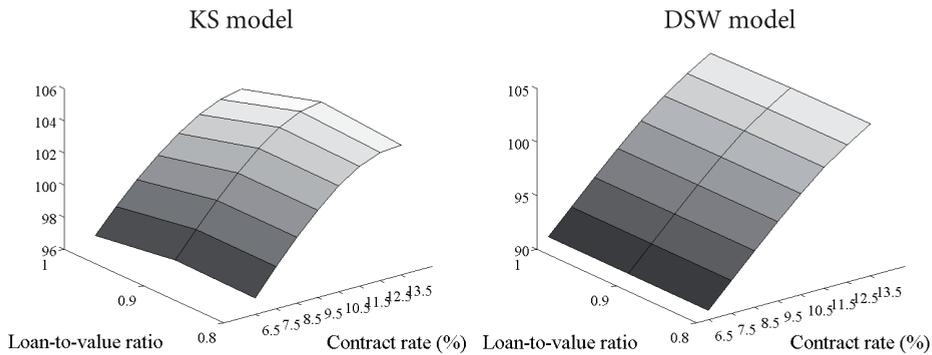
options values, PVRP, is directly affected by the change in  $\sigma_r^{20}$ , while it is not affected by changes in values of  $\sigma_H$  or LTV.

It is also observed that increasing the LTV ratio decreases the value of the MBS, due to its direct effect on the default option value.

*Sensitivities to the contract rate and LTV ratio*

The MBS value increases in contract rate for all loan-to-value ratio levels. High values of loan-to-value ratio induce the default option to be deep in the money and it takes a small change to exercise it. As a result its value rises with an increase in LTV. A similar result holds for the prepayment option in the case of higher contract rates. This effect is transferred by averaging of the values of individual contracts to the value of MBS.

**Figure 3. Variation in the values of c and LV of MBS value**



Note: The calculations were done using the following parameters: servicing fee 0.5%, maturity 5 years, service flow 8.5%, steady-state spot rate 10%, speed of adj. 0.25, interest rate st.dev. 10%, house price st.dev. 10%, no correlation, three levels of transaction cost.  $\lambda$  parameters for DSW model:  $\beta_0=0.1, \beta_1=1.5, \beta_2=0.4, \beta_3=1.1$ .

It can be observed in the case of the KS model that for a fixed (higher) level of contract rate the MBS value increases to a certain point and then decreases, with an increase in LTV. This can again be explained by the behaviour of the options. The prepayment option value is in the money for the high value of contract rate. With increasing LTV, the default option increases and forces a decline in

20 This is due to the effect on the extent of the inverse relationship between interest rate and PVRP. The gains on interest rate decrease are higher than the losses on interest rate increase of the same magnitude, resulting in higher PVRP with increasing volatility.

the prepayment option value. The rising value of the default option forces the borrower to wait longer with the exercise of the prepayment option, inducing more monthly payments coming to the mortgage holder at a higher contract rate, leading to a higher mortgage value. However, when the LTV is sufficiently high, both options are deep in the money and the mortgage value drops due to the high probability of termination at favourable contract rates. For lower levels of contract rate, as the prepayment option is not in the money, this competing effect of options is largely offset. The increasing loan-to-value ratio increases the default option to a lesser extent resulting in an overall smaller decrease in the mortgage value at high values of LTV.

This behaviour is not observed in the DSW model, which is explained by its dependence on the exogenous termination, and inability to incorporate the interrelations of the underlying mortgage component in the various economic environments.

### 3.2. MBS pricing, duration and convexity

Table 5 represents prices, effective duration, and effective convexity for a hypothetical MBS with respect to the initial spot rate and the assumed set of parameters. It can be observed that the respective MBS is priced in line with expectations. For interest rate level near the coupon rate, the MBS is almost priced at par. For lower interest rate values, the respective MBS is priced as a premium security, while for higher interest rate levels it is priced as a discount. It can also be seen that MBS exhibits negative convexity<sup>21</sup>. The table demonstrates that both duration and convexity decrease with a lower level of interest rate, as prepayments then become more likely.

**Table 5. MBS price, duration and convexity**

KS model				DSW model			
$r_0$	Price	Duration	Convexity	$r_0$	Price	Duration	Convexity
6%	103.10	0.67	36.18	6%	104.23	1.23	-2.31
8%	101.86	1.25	-82.98	8%	102.10	1.53	-30.23
10%	99.46	1.76	-13.66	10%	99.40	1.96	-24.79
12%	96.78	1.93	-13.80	12%	96.49	2.11	-2.23
14%	94.01	2.02	1.99	14%	93.58	2.18	4.07

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, steady-state spot rate 10%, interest rate volatility

<sup>21</sup> This is a common characteristic of MBS, due to the opposite effect of prepayments and the discount factor when interest rates decline, as prepayments lower the price increase.

10%, speed of adjustment 0.25, house price volatility 10%, loan-to-value ratio 90%, and there are no correlation between the two sources of risk. Parameters  $\lambda$  for DSW model are:  $\beta_0=0.1$ ,  $\beta_1=1.5$ ,  $\beta_2=0.4$ , and  $\beta_3=1.1$ .

In order to quantify the influence of default option value on MBS valuation, we ran the simulations in a set up where the default option is excluded from the algorithm, meaning that the possible choices for the borrower are only continuation or prepayment. Frictions are included as before.

**Table 6. MBS and mortgage assets values for different transaction costs with and without default option**

KS model									
with option to default					without option to default				
$r_0$	MBS	MBS1	MBS2	MBS3	$r_0$	MBS	MBS1	MBS2	MBS3
6%	101.310	99.689	99.822	104.420	6%	103.000	101.270	101.270	106.450
8%	100.130	99.060	99.450	101.870	8%	102.210	101.210	102.170	103.230
10%	98.214	97.624	97.774	99.244	10%	100.080	100.070	100.040	100.140
12%	95.912	95.545	95.611	96.580	12%	97.171	97.203	97.149	97.160
14%	93.478	93.238	93.277	93.920	14%	94.304	94.327	94.291	94.292

DSW model									
with option to default					without option to default				
$r_0$	MBS	MBS1	MBS2	MBS3	$r_0$	MBS	MBS1	MBS2	MBS3
6%	104.100	102.940	103.160	106.210	6%	104.220	103.030	103.220	106.420
8%	102.220	101.750	102.060	102.850	8%	102.400	101.900	102.310	102.990
10%	99.475	99.437	99.377	99.610	10%	99.671	99.690	99.634	99.690
12%	96.413	96.413	96.352	96.476	12%	96.543	96.591	96.517	96.522
14%	93.420	93.426	93.380	93.454	14%	93.499	93.538	93.480	93.480

Note: The calculations were done using the following parameters: contract rate 10.5%, servicing fee 0.5%, maturity 5 years, service flow 8.5%, steady-state spot rate 10%, interest rate volatility 5%, speed of adjustment 0.25, house price volatility 20%, loan-to-value ratio 95%, and no correlation between the two sources of uncertainty. KS model:  $\lambda=2*PSA$ .  $\lambda$  parameters for DSW model:  $\beta_0=0.1$ ,  $\beta_1=1.5$ ,  $\beta_2=0.4$ ,  $\beta_3=1.1$ . MBS<sub>i</sub>,  $i=1,2,3$  are mortgage assets values for low, medium and high transaction costs.

Table 6 presents the results for the MBS value and mortgage asset value for the three different levels of transaction costs<sup>22</sup> in the two scenarios - when default option is part of the valuation framework and when it is excluded. The parameter values were chosen to favour default – higher values of loan-to-value ratio and

<sup>22</sup> Denoted by MBS1 that relates to borrowers of the lowest transaction const, MBS2 medium, and MBS3 the highest.

house price volatility. We see that exclusion of the default option raises the MBS value in the chosen economic environment in both models. The effect of MBS value increase is lower in the case of the DSW model, due to its lower ability to respond to underlying variables movements. It should be noted that the total value of the default option is understated, as its value is reduced when joined with the prepayment option. Equivalently, the prepayment option in the framework without the default option is overstated, as it does not compete with the default for value.

#### **4. CONCLUSION**

In this paper, we extend the work of Hilliard, Kau and Slawson (1998) and Kau and Slawson (2002) in order to value MBS securities. We do so by introducing the heterogeneity of borrowers in the manner of Downing, Stanton and Wallace (2003). Through various simulations we find that the models are flexible enough to capture most of the known features that MBSs exhibit.

It should be noted that the adopted approach is in a rather theoretical framework and no input parameters are calibrated to actual data, but are chosen in line with the presented literature. However, calibrating the model to the actual data would be one of the important extensions of this work, which could possibly lead to certain modifications.

The model aims to describe how the interaction of different components of the mortgage contract and economic environment affect MBS value. The presented models account for many different determinants of borrowers' behaviour and are thus adaptive to different economic scenarios and can capture the influence of various exogenous factors. Furthermore, we have seen that the DSW model is less sensitive to the movements of the underlying variables due to its dependence on exogenous termination that is only partly connected to the underlying variables.

As seen from the simulations, different components such as house price level, house price volatility, loan-to-value ratio, value of the prepayment option and its determinants (as prepaying leaves less opportunity for defaulting and vice versa), all affect default of the individual mortgage contract, and thus the value of the MBS. In certain economic environments the value of the default option is even larger than and more influential on the overall value of the mortgage than the prepayment option. The recent crisis seems to confirm this view.

## REFERENCES

- Bardhan, A., Karapadža R. & Urošević, B. (2006). Valuing mortgage insurance contracts in emerging market economies. *Journal of Real Estate Finance and Economics*, 32(1), 9-20.
- Cox J., Ingersoll, J. & Ross, S. (1985). A theory of term structure of interest rates. *Econometrica*, 53(2), 385-407.
- Drenovak, M. & Urošević, B. (2010). Modelling the benchmark spot curve for the Serbian market. *Economic Annals*, 55(184), 29-57.
- Downing, C., Stanton, R. & Wallace, N. (2003). *An empirical test of a two-factor mortgage valuation model: How much do house prices matter?*. (Finance and Economics Discussions Series Working Paper No. 2003-42). Retrieved from the Board of Governors of the Federal Reserve System website: <http://www.federalreserve.gov/pubs/feds/2003/200342/200342pap.pdf>
- Fabozzi, F. J. (2004). *Fixed income analysis for the chartered financial analyst® program*. 2nd Ed. New Hope, PA: Frank J. Fabozzi Associates
- Goncharov, Y. (2006). An intensity-based approach to the valuation of mortgage contracts and computation of the endogenous mortgage rate. *International Journal of Theoretical and Applied Finance*, 9(6), 889-914.
- Hendershott, P. H. & Van Order, R. (1987). *Pricing mortgages: an interpretation of the models and results*. (NBER Working Paper No. 2290). Retrieved from the National Bureau of Economic Research website: <http://www.nber.org/papers/w2290.pdf>
- Hillard, J.E., Kau, J.B. & Slawson, V.C. (1998). Valuing prepayment and default in a fixed-rate mortgage: A bivariate binomial options pricing technique. *Journal of Real Estate Economic*, 26(3), 431-468.
- Hillard, J.E. & Schwartz, A. (1996). Binomial option pricing under stochastic volatility and correlated state variables. *Journal of Derivatives*, 4, 23-34.
- Kalotay, A., Yang, D. & Fabozzi, F.J. (2004). An option-theoretic prepayment model for mortgages and mortgage-backed securities. *International Journal of Theoretical and Applied Finance*, 8, 949-978.
- Kau, J.B. et al. (1992). A generalized valuation model for fixed-rate residential mortgages. *Journal of Money, Credit and Banking*, 24(3), 279-299.
- Kau, J.B. & Keenan, D.C. (1995). An overview of the option-theoretic pricing of mortgages. *Journal of Housing Research*, 6(2), 217-244.

- Kau, J.B. & Slawson, V.C. (2002). Frictions, heterogeneity and optimality in mortgage modelling. *Journal of Real Estate Finance and Economics*. 24(3), 239-260.
- Kolbe, A. (2007). *Valuation of mortgage products with stochastic prepayment-intensity models*. Technische Universität München
- Longstaff, F.A. (2002). *Optimal recursive refinancing and the valuation of mortgage-backed securities*. Retrieved from the eScholarship Repository, University of California website: <http://escholarship.org/uc/item/19k7479t>
- Nelson, D.B. & Ramaswamy, K. (1990). Simple binomial processes as diffusion approximations in financial models. *Review of Financial Studies*, 3(3), 393-430.
- Richard, S.F. & Roll, R. (1989). Prepayments on fixed-rate mortgage-backed securities. *Journal of Portfolio Management*, Spring 1989, 73-82.
- Schwartz, E.S. & Torous, W.N. (1989). Prepayment and valuation of mortgage-backed securities. *The Journal of Finance*, 44(2), 375-392.
- Schwartz, E.S. & Torous, W.N. (1992). Prepayment, default and the valuation of mortgage pass-through securities. *The Journal of Business*, 65(2), 221-239.
- Salomon Smith Barney. (2001). *Guide to mortgage-backed and asset-backed securities*, New York: John Wiley & Sons.
- Spahr, R.W. & Sunderman, M.A. (1992). The effect of prepayment modelling in pricing mortgage-backed securities. *Journal of Housing Research*, 3(2), 381-400.
- Stanton, R. (1995). Rational prepayment and the valuation of mortgage-backed securities. *Review of Financial Studies*, 8, 677-708.

Received: September 28, 2010

Accepted: October 27, 2010