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OPTIMAL TAX POLICY IN AN ENDOGENOUS GROWTH MODEL WITH A CONSUMABLE SERVICE GOOD

ABSTRACT: *The paper analyses the optimal tax policy in an endogenous growth model in a command economy, where the commodity output is produced with only physical capital, and skilled labour is the only input in producing the service good. In the benchmark model, per capita government expenditure is used to create human capital. Two cases are considered regarding taxation: in the first, tax is imposed on both commodity and service sectors, while in the second only the service sector is taxed. In each case the model derives the optimal tax rate and steady-state growth paths. In the first regime where both sectors are taxed we find the optimal tax rate on the service sector to be zero, but on commodity output it is positive. However, in the second regime there is also a unique optimal tax rate on the service sector to finance human capital accumulation. Comparing the growth rates in both cases we also observe that the imposition of tax on only the service sector instead of on both sectors yields a higher rate of growth if the population growth rate along with the marginal productivity of human capital is sufficiently high. We also show that when the service sector is taxed it may grow at a higher rate than the manufacturing sector. An extension of the benchmark model in which government spends tax revenue on accumulation of human capital as well as physical capital confirms that the optimal service tax rate is zero, but the optimal commodity tax rate is positive when both sectors are taxed.*

KEY WORDS: *taxation, government policy, endogenous growth, command economy, human capital accumulation.*

JEL CLASSIFICATION: E60, H21, O41

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1. INTRODUCTION

The choice of an optimal tax policy is an important issue in growth literature. The present paper focuses on the government's optimal tax policy in the presence of a consumable service sector. The development of endogenous growth theory has enabled policymakers to discover optimal fiscal policies in the growth model. To address the problems of optimal taxation a few assumptions are made. Following Slemrod (1990), it is assumed that all taxpayers are identical, so the government need not be concerned with the issues of vertical or horizontal equity. It is further assumed that tax rates can be raised without administrative cost. The basic problem of optimal tax policy literature is how to determine the tax rate that will maximize the economic growth rate, social welfare, or government revenue and leave the taxpayer as well off as possible. Around sixty years ago Ramsey (1927) showed that a uniform commodity tax system, which alters none of the relative prices of goods, was not optimal in general. Instead, the tax rate is negatively related to the consumption growth rate. Imposition of a lump-sum tax on the representative taxpayer is the first-best solution for optimal taxation, as the required revenue can be achieved without any efficiency cost. When there are very few restrictions, commodity taxation is optimal. The second-best solution for optimal taxation is the benign rule of uniform taxation (Slemrod1990). Slemrod(1990) carried out a detailed survey of existing literature on optimal tax policy.

There is a vast literature that also discusses the effects of various policies in endogenous growth models. Papers by Corlett and Hague(1953), Auerbach (1979) , Garcia-Castrillo and Sanso (2000), Kleven et.al(2000), Gomez (2003), Futagami, Morita, and Shibata (1993), Faig (1995), Dasgupta (1999), Chang (1999), Fernandez, Novales, and Ruiz (2004), Woo (2005), Chen and Lee (2007), Hollanders and Weel (2003), Greiner (2006) etc. shed light on this area. Given this background, the present study will try to find the government's optimal tax policy in an endogenous growth model in an economy with a consumable service good under the assumption that the government spends tax revenue to finance public education in order to accumulate human capital –similarly to Capolupo (2000),who investigates the long-run effects of government spending and taxation in an endogenous growth framework in which government spending on public education drives human capital accumulation. Several other papers consider the role in endogenous growth models of tax-financed government expenditure on public resources. Garcia-Castrillo and Sanso (2000) and Gomez (2003) design optimal fiscal policies in the Lucas (1988) model. Gomez (2003) also finds that atax-financed educational subsidy policy is optimal. However, Gomez' analysis (2003) does not finda lump sum tax to be optimal to finance the subsidy. Greiner (2008) examines the effects of fiscal policy in an endogenous growth model with two types of agent, skilled and unskilled, where human capital accumulation is a function of existing human capital the educational sector and public spending on education. The paper shows that a welfare-maximizing labour-income tax rate might exist even when a higher labour-income tax rate always raises the balanced growth rate.

Across the world, human capital and the education sector play a very important role in economic development and a lot of work has been done on the theory of human capital accumulation in growth economics. Hollanders and Weel (2003)address the role of public expenditure in human capital accumulation in a Lucas-type (1988) growth model. Greiner (2006) focuses on an endogenous growth model which is based on the assumption that human capital accumulation results from the investment of public resources, financed by imposing an income tax and issuing government bonds. Following Heckman (1976) and Rosen (1976), King and Rebelo (1990) try to discover the optimal accumulation of human capital and the effect of various taxation regimes on optimal accumulation. Their basic finding is that the cost of taxation on welfare is higher for endogenous growth models than in neoclassical models with exogenously given technical progress. The only study that deals with the service sector and optimal taxation is by Kleven et al.(2000). They find that market-produced services that are close substitutes for home-produced services should be given preference in an optimal tax system. The finding of this paper amends the classical Corlett-Hague (1953) rule for optimal commodity taxation and reveals that imposition of a lower tax rate on consumer services may be optimal in spite of the complementarily between services and leisure. Further, the study finds that when leisure can be equally substituted by services and other goods it is optimal to tax commodities uniformly, and when home production is absent the optimal tax structure will include a lower rate of tax on consumer services.

However, none of these growth models addresses the problems of the service sector along with the human capital accumulation function or discusses whether or not it is important to levy tax on service goods. In this paper, human capital is considered one of the most important factors in producing service output. Most of these services –education, health, public administration, banks, computer services, recreation, communications, financial services,and many others –require specialized know-how. A number of papers[1] have dealt with the importance of human capital in the service sector. Abowd et.al (2001) note that in service industries the service is fundamentally delivered by human capital.Kianto Hurmelinna-Laukkanen (2010) also find that service-oriented companies possess more human capital and renewal capital than product-oriented companies and focus more on intellectual capital creation. Messina (2004) considers the service sector to be characterized by relatively skill-oriented human-capital-intensive production compared to the manufacturing sector. According to an OECD(2000) study, the service sector employs a much higher share of university-educated workers than the goods sector, and in 1998 the ratio of university to non-university workers engaged in service industries was 0.24, whereas in the manufacturing sector the same ratio was .07. A briefing note by the Department for International Development (UK2008) also emphasizes the importance of human capital accumulation, saying that countries need to expand the human capital base in those professions whose services they are likely to export. Maroto Sánchez(2010) also acknowledges the role of human capital in the service sector. Simões and Duarte (2014), in the context of Portugal, show that modern progressive services register higher productivity levels and growth but demand higher levels of human capital to expand. Fang and Chao(2015) provide a detailed literature survey showing the positive impact of human capital in the development of tertiary industry in China. In their study of Shandong Province they show that the stock and level of human capital contributes positively to the development of tertiary industry. Thus, the literature on the service sector documents the importance of human capital as a service sector input.

Given this background, in the present paper we assume the existence of a hypothetical command economy where the only service sector input is human capital, which is accumulated through per capita government expenditure on the education sector. This is our benchmark model. Physical capital is used to produce commodity output only. Government expenditure is financed by tax revenue. We consider two tax regimes. In the first, tax is imposed on both consumption goods and service goods, while in the secondonly the service sector is taxed. We compare the steady-state growth paths of the two types of tax regime to discover the optimal tax policy. We also extend the model in order to consider the situation where government spends to accumulate human capital as well as physical capital in the above-mentioned tax regimes.

[1] Riley, 2012; Lucas, 1988; Mankiw et al., 1992; De La Fuente and Domenech, 2006

We find that in the benchmark model of a command economic regime, both human capital and commodity output have a positive, unique steady-state growth rate. In the first case where we consider tax on both the sectors, we find that the optimal tax rate for the service sector is zero, but the optimal tax rate for commodity output is positive. However, in the second case where tax revenue comes solely from a service tax, we find that there is a unique optimal tax rate for the service sector to finance human capital accumulation. Comparing the growth rates in both cases we also observe that taxing only the service sector instead of both sectors yields a higher rate of growth if the population growth rate along with the marginal productivity of human capital is sufficiently high. We also find that when the service sector is taxed to finance human capital accumulation it may grow at a higher rate than the manufacturing sector. In the case of the extended model where a fraction of government tax revenue is spent on human capital accumulation and another fraction goes to physical capital accumulation, when both sectors are taxed as in the benchmark model, the optimal tax on the service sector is still found to be zero. We also find that steady-state growth is the same under both tax regimes.

In this paper we are trying to find the optimal indirect tax on final goods and services. Existing theoretical papers on optimal indirect taxation show that final goods and services should be taxed uniformly, exempting intermediate goods. However, in practice value added tax, which is typically imposed on final goods and services, is imposed at different rates. In this paper, in a simplified model where human capital is used only to produce final services, while physical capital is used as the only input to produce final commodities, we offer an alternative theory of optimal policy. We suggest that the optimal policy should impose tax only on final commodity and not on services, irrespective of whether tax revenue sponsors only human capital accumulation or both human and physical capital accumulation. However, if tax is only imposed on services, even though it is not optimal, the economy may grow at a faster rate.

The rest of the paper is organised as follows. In section 2 the basic general model is presented. Section 3 discusses the optimal tax policy and steady-state growth paths and corresponding comparative static results under a command economic regime when tax is levied on both service and commodity sectors, modifies the model under the assumption that only service goods are being taxed, and comparesgrowth rates under the two tax regimes to find the optimal tax policy. Section 4 describesan extension of the benchmark model where tax revenue is spent on accumulation of human capital as well as physical capital. Section 5 concludes.

2. THE MODEL

This section describes the basic model for the functioning of an economy under a command economic regime.

Households, Firms, and Government

A closed economy model is considered that has two sectors, a commodity sector and a service sector. We assume that the output of the commodity sector is used for consumption or investment. The output of the service sector is fully consumed. The total labour force is homogeneous as far as skill is concerned. The commodity and the factor markets are characterized by perfect competition. Identical rational agents inhabit the economy. Production technology is subject to constant returns to scale. Preferences for the consumption of different combinations of commodity and service output are given by the following function, where *c* and *s* denote the flow of real per capita consumption of commodity and service outputs respectively.[2]

$$u(c) = \int_0^{\infty} \frac{(c^{\alpha} s^{1-\alpha})^{1-\sigma} - 1}{(1-\sigma)} e^{-\rho t} N(t) dt \tag{1}$$

Here, α is the parameter that measures the intensity of preference for commodity consumption and $(1-\alpha)$ measures the preference for service output consumption. Let ρ be the discount rate and σ the elasticity of marginal utility, the inverse of which is known as inter-temporal elasticity of substitution. Let N represents the total labour force or working population.

The commodity and service output production functions can be written as

$$y_c = AK \tag{2}$$

Following Das and Saha (2015) we assume

[2] Following Lucas (1988), we include *N(t)* in the utility function.

$$y_s = B(Nh) \tag{3}$$

where y_c and y_s are commodity and service outputs. K is the aggregate physical capital. Let the general skill level of a worker be h. The effective skilled work force in commodity production is Nh. A and B are the production technology parameters of commodity and service production functions that give the average as well as marginal productivity of each factor in the AK-type production function.

The population level is growing at an exponential rate in the following manner:

$$N(t) = N_0 e^{nt} \tag{4}$$

Here, N_0 stands for the population size during the initial time period. For simplicity, the initial population size is normalised, i.e., N_0=1. According to our assumption, government spends money on education to create human capital.

Following Glomm and Ravikumar (1992), Capolupo (2000), and Beauchemin (2001), we assume that human capital accumulation takes place through full-time public education. The human capital accumulation function can be written as

$$\dot{h} = \eta \frac{G}{N} \tag{5}$$

Here η is the technology parameter of human capital accumulation whose value is always positive, and G stands for government expenditure.

3. GROWTH RATES IN THE BENCHMARK MODEL UNDER DIFFERENT TAX REGIMES

3.1 Imposing tax on both commodity and service sectors

In this section we assume that both the commodity sector and the service sector are being taxed to finance government expenditure to build human capital. Let the tax rate levied per unit on service output production be τ_s and the tax rate imposed per unit on commodity output production be τ_k. Let the per unit price of service output be p_s and that of commodity output be 1. The balanced budget equation can be written as

$$G = T = p_s \tau_s y_s + \tau_K y_c \tag{6}$$

It is assumed that the disposable commodity output over aggregate consumption is accumulated as physical capital. The physical capital accumulation function is given by

$$\dot{K} = (1 - \tau_k) y_c - Nc \tag{7}$$

Equation (7) also satisfies the feasibility constraint. The demand for production of goods is demand for consumption goods (*Nc*) and for service goods (*Ns*), investment demand ($\dot{K}$), and government demand for goods over tax revenue. On the other hand the supply of goods is commodity production and service production. Since theentire disposable service good is consumed and the government runs a balanced budget, equation (7) equals the feasibility constraint. We assume there is no depreciation of physical capital and human capital. The per unit price of service output p_s is obtained from the consumer's equilibrium condition by equating the marginal rate of service product substitution for commodity output with the ratio between service output price and commodity output price.

$$p_s = \frac{(1-\alpha)}{\alpha} . \frac{(\frac{c}{h})}{(1-\tau_s)B} \tag{8}$$

For simplicity, it is assumed that the commodity output is a numeraire good, which implies that the per unit price of the commodity output, i.e., p_c, should be 1.

After deducting the taxable amount, the remaining disposable service output value is equal to the expenditure incurred on service goods, as the service goods are totally consumed by the population. So the market clearing condition is

$$p_s(1-\tau_s) y_s = p_s s N \tag{9}$$

The social planner maximizes the present discounted utility value over the infinite time horizon defined by equation (1), subject to the constraints of physical and human capital.

In this case, the control variables are c, τ_s, τ_K and the state variables are K and h. The current value Hamiltonian function is formulated as

$$H = N(t)\left[\frac{(c^{\alpha}s^{1-\alpha})^{1-\sigma}-1}{(1-\sigma)}\right]+\theta_1\dot{K}+\theta_2\dot{h}$$

Substituting the value of the physical capital and human capital investment functions,we get

$$H = N(t)[\frac{[(c^{\alpha}s^{1-\alpha})^{1-\sigma}-1}{(1-\sigma)}]+\theta_1[(1-\tau_K)y_c-cN]+\theta_2\eta\frac{(p_s\tau_s y_s+\tau_K y_c)}{N} \tag{10}$$

where θ_1 and θ_2 are the shadow prices associated with $\dot{K}$ and h, which stand for physical capital investment and human capital accumulation.From the first-order conditions of the control variables and the two co-state equations, the growth rate of commodity output consumption and that of human capital accumulation and physical capital are solved along with the tax rates. The first-order conditions are:

$$\alpha c^{\alpha(1-\sigma)-1}(1-\tau_s)^{(1-\alpha)(1-\sigma)}B^{(1-\alpha)(1-\sigma)}h^{(1-\alpha)(1-\sigma)}N(t)-\theta_1 N(t)+\theta_2\eta\frac{(1-\alpha)\tau_s}{\alpha(1-\tau_s)}=0 \tag{11}$$

$$\theta_1=\frac{\theta_2\eta}{N} \tag{12}$$

$$N(t)c^{\alpha(1-\sigma)}B^{(1-\alpha)(1-\sigma)}h^{(1-\alpha)(1-\sigma)}(1-\alpha)(1-\tau_s)^{(1-\alpha)(1-\sigma)-1}+\theta_2\eta\frac{(1-\alpha)}{\alpha}c\frac{d}{d\tau_s}(\frac{\tau_s}{1-\tau_s})=0 \tag{13}$$

$$\dot{\theta}_1=\rho\theta_1-\left\{\theta_1(1-\tau_K)A+\theta_2\frac{\eta\tau_K}{N}(\frac{A}{N})\right\} \tag{14}$$

$$\dot{\theta}_2=\rho\theta_2-\left[N(t)c^{\alpha(1-\sigma)}(1-\tau_s)^{(1-\alpha)(1-\sigma)}B^{(1-\alpha)(1-\sigma)}(1-\alpha)h^{(1-\alpha)(1-\sigma)-1}\right] \tag{15}$$

Steady-state growth path and tax rate:

Proposition 1.

Under a command economic regime, human capital and consumption of commodity output and physical capital exhibit a positive, unique stead-state growth rate when tax is imposed on both sectors. Further, the optimal tax rate for the service sector is zero while that for commodity output is positive.

Proof:

(For detailed proof see Appendix 1.)

From first order conditions, we obtain the steady-state growth rate of*c, h, and K* and the uniquely determined values of tax rates.

$$\gamma_h = \gamma_c = \frac{(A-\rho)}{\sigma} > 0 \ A > \rho \tag{16}$$

$$\gamma_K = \gamma_h + n \tag{17}$$

$$\tau_K = \frac{(1-\alpha)(A-\rho)[\sigma(A-n)-(A-\rho)]}{A\sigma[(1-\alpha)(A-\rho)+\sigma\alpha(A-n)]} \tag{18}$$

$$\tau_s = 0 \tag{19}$$

$$k = \frac{K}{Nh} = \frac{(A-\rho)}{\sigma\tau_K A\eta} > 0 \Rightarrow A > \rho \tag{20}$$

Thus, when the rate of time preference (ρ) is lower than the marginal productivity of capital(A), the capital–labour ratio is always positive,[3] and this in turn implies that the economy grows at a positive rate at the steady state. From the investment function in equation (7) it is clear that the disposable income that is left after consuming the commodity output is invested in creating physical capital. As the future discount rate falls, individuals become more concerned with future consumption rather than present consumption. Therefore, more investment foregoes present consumption. As a result, future

3 This is the standard assumption of the AK model.

consumption increases and present consumption decreases. Hence, the consumption growth rate and the output growth rate increase.

Further, $(\frac{c}{h}) = \frac{(A-n)}{\eta}\frac{\alpha}{(1-\alpha)} > 0 \Rightarrow A > n$ (21)

As the ratio of per capita commodity consumption to per capita skill level can never be negative, $A>n$. This condition for the positive value of per capita commodity consumption and the general skill level ratio require that the value of marginal capital productivity should be greater than the population growth rate. When the population growth rate is low, more output is left to be consumed by fewer individuals. This raises the commodity-consumption-to-skill ratio.

Equations (20) and (21) together with the condition $\sigma(A-n)>(A-\rho)$ imply that $0<\tau_K\leq 1$.

From equation (19), we observe that it is not optimal to tax the service sector. This result is very interesting. Auerbach (1979) explores the issue of optimal capital taxation and finds the optimal capital tax to be zero. Alternatively, Kleven et al.(2000) find that imposing a lower tax rate on consumer services may be optimal in spite of the complementarity between services and leisure. In our present paper, where household utility depends on commodity consumption and services consumption, we observe that if there is a possibility to tax commodity sector as well as service sector, it is optimal to tax only the commodity sector and not the service sector.

Next, we check the comparative static results corresponding to various parameters.

Differentiating the optimal commodity tax rate as given in (18) with respect to the inverse of inter-temporal elasticity of substitution, we find that

$$\frac{d\tau_K}{d\sigma} = \frac{A(A-\rho)(1-\alpha)[A(1-\alpha)(A-\rho)^2 + \alpha\sigma(A-n)\{2(A-\rho)-\sigma(A-n)\}]}{\{A\sigma[(A-\rho)(1-\alpha)+\sigma\alpha(A-n)]\}^2}$$

The sufficient condition for $\frac{d\tau_K}{d\sigma} > 0$ is $A > \frac{2\rho - n\sigma}{2-\sigma}$. If the condition is not satisfied $\frac{d\tau_K}{d\sigma}$ may be positive or negative.

Further differentiating (16) with respect to σ gives $\frac{d\gamma_h}{d\sigma} < 0$

In this case, the growth rate of human capital γ_h always decreases for an increase in σ, but its impact on the optimal tax rate is ambiguous.

Differentiating (18) with respect to the rate of discount, we get

$$\frac{d\tau_K}{d\rho} = \frac{A\alpha\sigma^2(A-n)(1-\alpha)[2(A-\rho)-\sigma(A-n)]+(1-\alpha)^2(A-\rho)^2 A\sigma}{\{A\sigma[(1-\alpha)(A-\rho)+\alpha\sigma(A-n)]\}^2}$$

If, $\frac{(A-\rho)}{(A-n)} < \sigma < \frac{2(A-\rho)}{(A-n)}$

then $\frac{d\tau_K}{d\rho} > 0$ as well as τ_K is positive.

And $\frac{d\gamma_h}{d\rho} < 0$

As the discount rate rises, individuals become more concerned about present rather than future consumption. Thus, under a balanced budget the tax rate will rise that raises the future accumulation rate of human capital to be used as an input to produce service output in subsequent periods, for a rise in ρ if $2(A-\rho) > \sigma(A-n)$. However, irrespective of the movement in the tax rate, the growth rate of human capital accumulation will fall along with the rising values of the rate of time preference.

We note that the conditions for $\frac{d\tau_K}{d\rho} > 0$ and $\frac{d\tau_K}{d\sigma} > 0$ are the same. These conditions are likely to be satisfied if the values of ρ and σ are low.

Differentiating (18) with respect to the intensity of preference towards commodity consumption we get

$$\frac{d\tau_K}{d\alpha} = \frac{A\sigma^2(A-\rho)(A-n)[(A-\rho)-\sigma(A-n)]}{\{A\sigma[(A-\rho)(1-\alpha)+\sigma\alpha(A-n)]\}^2}$$

If $\tau_K > 0,\ A > \dfrac{\sigma n-\rho}{\sigma-1}$

Therefore, $\dfrac{d\tau_K}{d\alpha} < 0$.

If individuals derive more utility from commodity consumption than service consumption it is optimal to reduce tax on commodity output. This result implies that as the intensity of preference toward service consumption increases, the value of τ_K rises.

3.2 When tax is imposed on only the service sector

In this section we consider the case where only the service sector is taxed. The tax revenue (equal to government expenditure) is spent to build human capital. Let the tax rate be τ_s, which is levied on per unit production of service output. Now the balanced budget equation can be written as

$$G = T = \tau_s y_s \tag{22}$$

In this model we assume that there is no depreciation of physical or human capital. Using equations (3), (5), and (6), the human capital accumulation function is given by

$$\dot{h} = \eta \frac{G}{N} = \eta \tau_s Bh \tag{23}$$

After deducting the taxable amount, the remaining disposable service output is totally consumed by the population. Thus the market clearing condition is derived as

$$s = (1-\tau_s)Bh \tag{24}$$

It is assumed that commodity output over aggregate consumption is accumulated as physical capital. The physical capital accumulation function is given by

$$\dot{K} = y_c - Nc \tag{25}$$

Equation (25) also satisfies the feasibility constraint, just like equation (7). The social planner in a command economy maximizes the value of utility defined by equation (1) subject to the physical capital accumulation and human capital accumulation constraints given by equations (23) and (25). The value of *s*, which denotes per capita consumption of service output in our model, is substituted by equation (24) in the following Hamiltonian function.

The current value Hamiltonian as given in (10) is maximised with respect to the control variables c and τ_s where the state variables are K and h. Here, θ_1 and θ_2 are the shadow prices associated with $\dot{K}$ and $\dot{h}$, which stand for physical capital investment and human capital accumulation.

$$H = N(t)[\frac{[c^{\alpha}\{(1-\tau_s)Bh\}^{1-\alpha}]^{1-\sigma} - 1}{(1-\sigma)}] + \theta_1[AK - cN] + \theta_2 \eta\tau Bh \tag{26}$$

The first order conditions for the maximization of present discounted value of utility are given by:

$$\alpha c^{\alpha(1-\sigma)-1}(1-\tau_s)^{(1-\alpha)(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} = \theta_1 \tag{27}$$

$$N(t)c^{\alpha(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\alpha)(1-\tau_s)^{(1-\alpha)(1-\sigma)-1} = \theta_2 Bh\eta \tag{28}$$

$$\frac{\dot{\theta}_1}{\theta_1} = (\rho - A) \tag{29}$$

$$\frac{\dot{\theta}_2}{\theta_2} = \rho - B\eta \tag{30}$$

Steady-state growth path and tax rate:

Proposition 2: ***In a command economy, human capital and commodity output havea positive, unique steady-state growth rate. There also exists a unique optimal tax rate on the service sector to finance human capital accumulation.***

(For proof see Appendix 2.)

From first order conditions we obtain the growth rates of commodity output and human capital

$$\gamma^*_c = \frac{[(1-\alpha)(1-\sigma)(n+B\eta)-\rho]}{\sigma} + \frac{A[\sigma+\alpha(1-\alpha)(1-\sigma)^2]}{\{1-\alpha(1-\sigma)\}\sigma} \tag{31}$$

$$\gamma_h^* = \frac{A\alpha(1-\sigma)+\{1-\alpha(1-\sigma)\}(n+B\eta)-\rho}{\sigma} \tag{32}$$

The value of optimal tax rate is

$$\tau_s^* = \frac{A\alpha(1-\sigma)+\{1-\alpha(1-\sigma)\}(n+B\eta)-\rho}{B\eta\sigma} \tag{33}$$

Now we compare the human capital growth rate and commodity output growth rate.

Proposition 3*:* ***When only the service sector is taxed it will grow at a higher rate than the commodity sector if the sum of the population growth rate and the technology parameter of the service sector is higher than the technology parameter of the commodity sector.***

The comparison of γ_h and γ_c gives the following result:

$$\gamma_h^* - \gamma_c^* = (n+\eta B - A) \tag{34}$$

It implies that $\gamma_h^* < \gamma_c^*$ if and only if

$$A > n + B\eta \tag{35}$$

The reverse will be the case if $n + B\eta > A$

The condition implies that if the sum of the population growth and the weighted value (with the coefficient of human capital in the service output production function) of the technology parameter of human capital is more than the marginal productivity of physical capital, the growth rate of human capital accumulation and hence the service sector growth rate will be higher than the growth rate of commodity consumption. The result is intuitively obvious. In the model, human capital is used in the service production function, whereas physical capital is used in commodity production. Thus, if human capital becomes more efficient than physical capital the service sector will grow at a higher rate than the commodity sector, and vice versa. In developing countries like India, the service sector often grows at a much higher rate than the manufacturing sector (Bosworth, Collins, and Virmani, 2006; Banga and Goldar, 2007). Zuleta and Young (2013) explain this in terms of innovation in the manufacturing sector. They argue that as labour-saving innovation takes place in the manufacturing sector the labour force shifts to the service sector and consequently the service sector outperforms the manufacturing sector in contributing to the overall growth of the economy. Das and Saha (2015) argue that differences in returns to scale between the two sectors and employment frictions in manufacturing explain why the growth rate of the service sector may be higher. Our model tries to explain this fact in terms of population growth rate and the efficiency parameters of both the sectors. Since some developing countries have a high population growth rate they experience a disproportionate increase in service sector growth compared to manufacturing sector growth. If the service sector is taxed, services become more expensive to consume and individuals consume more commodity outputs, depleting physical capital and reducing manufacturing sector growth. Thus, taxing the service sector may yield unbalanced growth.

Next, we try to find how the tax rate and growth rates vary with respect to changes in different parameter values.

Differentiating the optimal service tax rate with respect to the inverse of inter-temporal elasticity of substitution, we find that

$$\frac{d\tau^*_s}{d\sigma} = \frac{-[(n+\eta B - A)(1-\alpha) + (A-\rho)]}{(B\eta\sigma)^2} \tag{36}$$

if $n + \eta B - A \geq 0$, and $A \geq \rho$, $\dfrac{d\tau_s}{d\sigma} \leq 0$.

As the inverse of inter-temporal elasticity of substitution, σ**,** falls, the optimal tax rate may increase if $A \geq \rho$ and $n+B\eta>A$. (See Appendix for detailed derivation.) This implies that as the inter-temporal elasticity of substitution increases (which is the same as a fall in σ), a representative consumer can easily substitute present consumption with future consumption. Thus, the present optimal tax rate for the service sector increases, because tax can finance education that can generate human capital in the future if $n+B\eta>A$ and $A\geq\rho$. Thus, when σ decreases, people are ready to forgo present consumption for future consumption and are willing to pay more tax when the service sector is more productive than the commodity sector and the time preference rate is not very high.

We now examine how the optimal service tax rate will respond when the technology parameter in human capital accumulation changes. Differentiating the optimal tax rate with respect to the technology parameter of human capital accumulation we find

$$\frac{d\tau^*_s}{d\eta} = -\frac{(B\sigma)}{(\eta B\sigma)^2}[(A-n)\{1-\alpha(1-\sigma)\}-(A-\rho)] \tag{37}$$

If $(A-n)\{1-\alpha(1-\sigma)\}<(A-\rho)$

Alternatively, $n > A - \dfrac{(A-\rho)}{\{1-\alpha(1-\sigma)\}}$

$$\frac{d\tau^*_s}{d\eta} > 0$$

If population growth is high, or above a critical level, then a rise in η, the technology efficiency parameter for human capital accumulation, may cause the tax rate to rise. If n is high, then per capita allocation of government spending on education is low. Thus, even when η rises, to increase the growth rate the optimal τ_s rises, and vice-versa.

Differentiating the optimal tax rate in a command economy with respect to the time preference rate, we find

$$\frac{d\tau^*_s}{d\rho} < 0$$

As the time preference rate rises, individuals become more concerned with present rather than future consumption. Thus, under a balanced budget, a tax rate increase that raises the future accumulation rate of human capital– which will be used as an input to produce commodity output in subsequent periods – will fall, against a rise in the time preference rate.

Now we will examine the changes in human capital accumulation and commodity consumption growth rates due to changes in different parameters.

The human capital growth rate is as follows:

$$\gamma^*_h = B\eta\tau_s$$

It is obvious from the human capital accumulation function that the response of γ^*_h to change in parameters α, ρ and σ, for example, will be same as the response of τ_s with respect to α, ρ and σ.

In this model the growth rate of human capital accumulation is positively related to the tax rate imposed on service output because the tax revenue is used to create human capital. Therefore, when the tax rate increases (decreases) due to an increase (decrease) of the parameters that we have considered in our analysis, the growth rate of human capital also increases (decreases) accordingly.

3.3 Comparison of growth rates

In this section we compare growth rates for the two different tax regimes discussed above.

Proposition 4: ***Taxing the service sector instead of taxing both sectors yields a higher growth rate if the population growth rate and the marginal productivity of human capital are sufficiently high. Otherwise, the reverse will happen.***

Proof: Comparing (16) and (32),we obtain

$$\gamma^*_h - \gamma_h = \frac{\{1-\alpha(1-\sigma)\}(n+B\eta-A)}{\sigma}$$

Therefore, the growth rate is higher when tax is imposed on the service sector rather than on both sectors if $n+B\eta>A$, and vice versa.

Note that this is the same condition we obtained for the service sector growing at a higher rate than the commodity sector. Thus, we can say that for $A < n + B\eta$, the service sector grows at a faster rate in the economy compared to when the commodity sector is taxed. Hence, imposition of tax on the service sector compared to taxing both sectors yields a higher growth rate. We know that A denotes the marginal product of capital in the commodity sector and B denotes the marginal product of human capital in the service sector. If the sum of the growth rate of the population and the marginal product of human capital in the service sector multiplied by the technology parameter in the human capital accumulation function ($B\eta$) is higher than A, then $\gamma_h^* > \gamma_h$. Thus, when the marginal product of capital to produce commodity output is relatively high, the economy experiences a higher growth rate if the commodity sector is taxed.

Finally, we consider a numerical example:

Let A be 1, B be 1, n be 0.05 and η be 1, then the growth rate when the service sector is taxed is higher than that when both sectors are taxed. But if we consider A to be 1, B to be 1, n to be 0.05, and η to be 0.5, then the growth rate when both sectors are taxed is higher than the growth rate when only the service sector is taxed.

4. EXTENSION OF THE BENCHMARK MODEL: GOVERNMENT SPENDS TO ACCUMULATE HUMAN AND PHYSICAL CAPITAL

In this section, we assume that government expenditure $G$is spent on accumulation of both human capital and physical capital. As a result, all other equations remaining unchanged, equations (5) and (7) are modified accordingly in the following way:

$$\dot{h} = \eta \frac{(\phi G)}{N} \tag{38}$$

and

$$\dot{K} = y_c + (1-\phi)G - Nc \tag{39}$$

Here, ϕ is the exogenously given fraction of G spent on human capital accumulation and $(1-\phi)$ is spent on physical capital accumulation. Here, G is given by Equation (6).

4.1.Government Expenditure is financed by taxing both commodity and service sectors

The current value Hamiltonian is given as[4]

$$H = N(t)[\frac{c^{\alpha(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\tau_s)^{(1-\alpha)(1-\sigma)} - 1}{(1-\sigma)}] +$$
$$\theta_1[(1-\tau_K)AK + (1-\phi)\tau_K(AK) + \tau_s N \frac{(1-\alpha)}{\alpha}(\frac{(1-\phi)c}{(1-\tau_s)}) - cN] \tag{40}$$
$$+ \theta_2 \eta \frac{\phi}{N}(\tau_K AK + \tau_s N \frac{(1-\alpha)}{\alpha}(\frac{c}{(1-\tau_s)}))$$

Here, the control variables are c, τ_K, and τ_s; K and h are the state variables and θ_1 and θ_2 are the co-state variables.

Using the first-order conditions of the optimization problem we find that

$$\tau_K = \frac{(1-\alpha)(A-\rho)[\sigma(A-n)-(A-\rho)]}{\sigma[(A+\phi-1)(1-\alpha)(A-\rho)+\sigma\alpha(A-n)\phi A]}$$

and $\tau_s = 0$. Note that if $\phi = 1$, the τ_K obtained in this extended model is same as that obtained in the basic model. We obtain that per capita consumption and human capital grow at the same constant rate given by $\gamma_c = \gamma_h = \frac{(A-\rho)}{\sigma}$.

Proposition 5: ***When government spends to accumulate human capital and physical capital and both sectors are taxed, under a steady state the optimal tax rate for services is τ_s=0 and for final commodities it is positive, and the steady-state growth rates are*** $\gamma_c = \gamma_h = \frac{(A-\rho)}{\sigma}$

(For proof see Appendix 3.)

4 Detailed derivation of the model is given in Appendix 3

Thus, as in the previous case where tax revenue is spent solely on human capital accumulation, in this case where tax revenue is spent on both human and physical capital accumulation, the optimal tax rate for service goods is zero while that for final commodities is positive.[5] As summarized by Mankiw et al. (2009), existing theoretical papers on optimal indirect taxation, namely Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1976), show that only final goods ought to be taxed and typically they ought to be taxed uniformly; taxes on intermediate goods should be avoided. Mankiw et al. (2009) also observe that in practice in many countries, value added taxes on goods and services that in principle exempt all intermediate goods are laden with exceptions and rules that violate the guidelines of optimal policy. This paper offers an alternative theory of optimal policy with a simplified model where human capital is used only in final services, while physical capital is the only input used to produce final commodities.

5. CONCLUSION

This paper constructs a two-sector endogenous growth model under a command economic regime in order to discover the optimal tax policy. Commodity output is produced with only physical capital, whereas skilled labour is the only input used to produce service output. Two tax regimes are considered. In the first regimeboth commodity goods and services are taxed. In the second regime only the service sector is taxed. We first consider the benchmark model where the tax revenue is invested to create human capital through government expenditure. Steady-state growth paths are studied under a command economic regime. The optimal tax rate and steady-state growth path are derived in each case. The growth rate is higher when tax is only imposed on the service sector rather than on both commodity and service sectors, provided the population growth rate and marginal productivity of human capital are sufficiently high. However, when both sectors are taxed the optimal tax on the service sector is zero while on commodity output it is positive. We also show that when the service sector is taxed it can grow at a higher rate than the manufacturing sector. Next, we extend the benchmark model to consider the case where tax revenue is spent on accumulating human capital as well as physical capital. In this case we also find that the optimal tax rate on final

5 In this extended model, if only the service sector is taxed, along the steady-state balanced growth path the optimal service tax is found to be positive. Derivations are available from the authors on request.

commodities is positive but that on services is zero. In this paper we are trying to discover the optimal indirect tax on final goods and services. Existing theoretical papers on optimal indirect taxation show that only final goods and services should be taxed at a uniform rate and taxes should avoid intermediate goods. However, across countries the value added tax rate imposed on final goods and services differ. This paper offers an alternative theory of optimal policy in a simplified model where human capital is used only in final services while physical capital is only used as an input to produce final commodities. This paper finds that the optimal tax on final commodities is positive, while that on final services is zero, whether tax revenue is spent on only human capital accumulation or on both human and physical capital accumulation. However, taxing only services may yield higher growth.

REFERENCES.

Abowd, J. M., Haltiwanger, J., Lane, J., & Sandusky, K. (2001). Within and between firm changes in human capital, technology, and productivity. Paper presented at 2002 SOLE Meeting, Baltimore, MD, draft (December 2001).

Atkinson, A. B., & Stiglitz, J. E. (1976). The design of tax structure: direct versus indirect taxation. *Journal of Public Economics*, *6*(1–2), 55–75.

Auerbach, A. J. (1979). The optimal taxation of heterogeneous capital. *The Quarterly Journal of Economics*, *93*(4), 589–612.

Banga, R., & Goldar, B. (2007). Contribution of services to output growth and productivity in Indian manufacturing: pre-and post-reforms. *Economic and Political Weekly*, 2769–2777.

Beauchemin, K. R. (2001). Growth or stagnation? The role of public education. *Journal of Development Economics*, *64*(2), 389–416.

Bosworth B., Collins S. M., and Virmani A. (2006). Sources of Growth in the Indian Economy. *India Policy Forum, Global Economy and Development Program, The Brookings Institution*, 3(1), 1–69.

Capolupo, R. (2000). Output taxation, human capital and growth. *The Manchester School*, *68*(2), 166–183.

Chang, W. Y. (1999). Government spending, endogenous labor, and capital accumulation. *Journal of Economic Dynamics and Control*, *23*(8), 1225–1242.

Chen, B. L., & Lee, S. F. (2007). Congestible public goods and local indeterminacy: A two-sector endogenous growth model. *Journal of Economic Dynamics and Control*, *31*(7), 2486–2518.

Corlett, W. J., & Hague, D. C. (1953). Complementarity and the excess burden of taxation. *The Review of Economic Studies*, *21*(1), 21–30.

Das, S. P., &Saha, A. (2015). Growth of business services: A supply-side hypothesis. *Canadian Journal of Economics/Revue Canadienned'Economique*, *48*(1), 83–109.

Dasgupta, D. (1999). Growth versus welfare in a model of nonrival infrastructure. *Journal of Development Economics*, *58*(2), 359–385

DFID (2008). The Contribution of Services to Development and the Role of Trade Liberalisation and Regulation.OECD Global Forum on International Investment OECD Investment Division www.oecd.org/investment/gfi-7. Accessed from *www.oecd.org/investment/globalforum/40302909.pdf*

Diamond, P. A., &Mirrlees, J. A. (1971). Optimal taxation and public production I: Production efficiency. *The American Economic Review*, *61*(1), 8–27.

Faig, M. (1995). A simple economy with human capital: transitional dynamics, technology shocks, and fiscal policies. *Journal of Macroeconomics*, *17*(3), 421–446.

Fernández, E., Novales, A., & Ruiz, J. (2004). Indeterminacy under non-separability of public consumption and leisure in the utility function. *Economic Modelling*, *21*(3), 409–428.

Futagami, K., Morita, Y., & Shibata, A. (1993). Dynamic analysis of an endogenous growth model with public capital. *The Scandinavian Journal of Economics*, 607–625.

De la Fuente, A., &Doménech, R. (2006). Human capital in growth regressions: how much difference does data quality make?*Journal of the European Economic Association*, 4(1), 1–36.

Garcia-Castrillo, P., & Sanso, M. (2000). Human capital and optimal policy in a Lucas-type model. *Review of Economic Dynamics*, *3*(4), 757–770.

Gómez, M. A. (2003). Optimal fiscal policy in the Uzawa-Lucas model with externalities. *Economic Theory*, *22*(4), 917–925.

Glomm, G., &Ravikumar, B. (1992). Public versus private investment in human capital: endogenous growth and income inequality. *Journal Of Political Economy*, *100*(4), 818–834.

Greiner, A. (2008). Human capital formation, public debt and economic growth. *Journal of Macroeconomics*, 30(1), 415–427.

Greiner, A. (2008). Fiscal policy in an endogenous growth model with human capital and heterogenous agents.*Economic Modelling*5(4): 643–657.

Heckman, J. J. (1976). A life-cycle model of earnings, learning, and consumption. *Journal Of Political Economy*,84(4, Part 2), S9–S44.

Hollanders, H. J. G. M., &terWeel, B. J. (2003). Skill-biased technological change: on endogenous growth, wage inequality and government intervention. In *Growth Theory And Growth Policy* (pp. 156–171). Routledge/Taylor & Francis Group.

Kianto, A., Hurmelinna-Laukkanen, P., &Ritala, P. (2010). Intellectual capital in service-and product-oriented companies. *Journal of Intellectual Capital*, *11*(3), 305–325.

King, R. G., & Rebelo, S. (1990). Public policy and economic growth: developing neoclassical implications. *Journal of Political Economy*, *98*(5, Part 2), S126–S150.

Kleven, H. J., Richter, W. F., &Sørensen, P. B. (2000). Optimal taxation with household production. *Oxford Economic Papers*, *52*(3), 584–594.

Lucas Jr, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, *22*(1), 3–42.

Mankiw, N. G., Weinzierl, M., &Yagan, D. (2009). Optimal taxation in theory and practice. *Journal of Economic Perspectives*, *23*(4), 147–74.

Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, *107*(2), 407–437.

Messina, J. (2005). Institutions and service employment: A panel study for OECD countries. *Labour*, *19*(2), 343–372.

Maroto Sánchez, A. (2010). Growth and productivity in the service sector: The state of the art. http://hdl.handle.net/10017/6558

OECD (2000) Employment in the Service Economy: a Reassessment. In *Employment Outlook*. Chapter 3. Paris. https://www.oecd.org/els/emp/2079561.pdf

Ramsey, F. P. (1927). A Contribution to the Theory of Taxation. *The Economic Journal*, *37*(145), 47–61.

Riley G. (2012). Economic Growth – The Role of Human & Social Capital, Competition & Innovation. http://www.tutor2u.net/economics/revision notes/a2-macro-economic-growth-capital.html Accessed April 16, 2014.

Rosen, S. (1976). A theory of life earnings. *Journal of Political Economy*, *84*(4, Part 2), S45–S67.

Simões, P. M. C., & Duarte, A. (2014). Human capital and growth in services economies: The case of Portugal. In *Structural Change, Competitiveness and Industrial Policy* (pp. 167–197). Routledge.

Slemrod, J. (1990). Optimal taxation and optimal tax systems. *Journal of Economic Perspectives*, 4(1), 157–178.

Woo, J. (2005). Social polarization, fiscal instability and growth. *European Economic Review*, 49(6), 1451–1477.

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APPENDIX 1

Proof of Proposition 1.

The first order conditions are

$$\frac{dH}{dc} = 0 \Rightarrow \alpha c^{\alpha(1-\sigma)-1}(1-\tau_s)^{(1-\alpha)(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} N(t) - \theta_1 N(t) + \theta_2 \eta \frac{(1-\alpha)\tau_s}{\alpha(1-\tau_s)} = 0 \quad \text{A1.}$$

$$\frac{dH}{d\tau_k} = 0 \Rightarrow \theta_1 = \frac{\theta_2 \eta}{N} \quad \text{A2.}$$

$$\frac{dH}{d\tau_s} = 0 \Rightarrow N(t)c^{\alpha(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\alpha)(1-\tau_s)^{(1-\alpha)(1-\sigma)-1} + \theta_2 \eta \frac{(1-\alpha)}{\alpha} c \frac{d}{d\tau_s}\left(\frac{\tau_s}{1-\tau_s}\right) = 0 \quad \text{A3.}$$

$$\dot{\theta}_1 = \rho\theta_1 - \left\{ \theta_1(1-\tau_K)A + \theta_2 \frac{\eta\tau_K}{N}\left(\frac{A}{N}\right) \right\} \quad \text{A4.}$$

$$\dot{\theta}_2 = \rho\theta_2 - \left[N(t)c^{\alpha(1-\sigma)}(1-\tau_s)^{(1-\alpha)(1-\sigma)} B^{(1-\alpha)(1-\sigma)} (1-\alpha) h^{(1-\alpha)(1-\sigma)-1} \right] \quad \text{A5.}$$

Manipulating A1 and A2 gives:

$$\alpha c^{\alpha(1-\sigma)-1}(1-\tau_s)^{(1-\alpha)(1-\sigma)+1} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} = \theta_1 \frac{(\alpha - \tau_s)}{\alpha} \quad \text{A6.}$$

From A3:

$$\frac{N(t)\alpha c^{\alpha(1-\sigma)-1} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\tau_s)^{(1-\alpha)(1-\sigma)+1}}{\eta} = \theta_2 \quad \text{A7.}$$

Using A2 in A4 we have

$$\frac{\dot{\theta}_1}{\theta_1} = (\rho - A) \quad \text{A8.}$$

Taking the log and differentiating A7 with respect to time *t*and using A2 and A6 we have

$$\frac{\dot{\theta}_2}{\theta_2} = \frac{\dot{\theta}_1}{\theta_1} + n = (\rho - A + n) \qquad \text{A9.}$$

Dividing A6byA7 and using A2 we have $(\alpha - \tau_s) = \alpha$ or

$$\tau_s = 0 \qquad \text{A10.}$$

Then equating A8andA9 and using A10

$$(\frac{c}{h}) = \frac{(A-n)}{\eta} \frac{\alpha}{(1-\alpha)} \qquad \text{A11.}$$

Since c/h>0, A>n.

This in turn implies $\gamma_h = \gamma_c$ A12.

Taking the logarithm of A6and using A8 gives

$$\gamma_h = \frac{(A-\rho)}{\sigma} \qquad \text{A13.}$$

From the model and using A10,

$$\gamma_h = \frac{\dot{h}}{h} = \eta \frac{G}{Nh} = \eta \frac{T}{Nh} = \tau_s p_s B(hN) + \tau_K (AK) = \tau_K A k \eta \qquad \text{A14.}$$

Thus, $k = \dfrac{(A-\rho)}{\sigma \tau_K A \eta}$ A15.

For k to be positive, the required condition is

$$(A-\rho) > 0$$

Thus $\gamma_h = \gamma_c = \frac{(A-\rho)}{\sigma} > 0$

Further from A14, it is clear that at the steady state, $k = \frac{K}{Nh}$ must be a constant.

Thus, $\gamma_K = n + \gamma_h$ A16.

From investment function

$$\frac{\dot{K}}{K} = (1-\tau_k)A - \left(c\frac{N}{K}\right) \text{ or}$$

$$\gamma_K = (1-\tau_K)A - \left(\frac{c}{kh}\right) \qquad \text{A17.}$$

Equating A16 and A17 and replacing A11, A14, and A15 we have

$$\tau_K = \frac{(1-\alpha)(A-\rho)[\sigma(A-n)-(A-\rho)]}{A\sigma[(1-\alpha)(A-\rho)+\sigma\alpha(A-n)]} \qquad \text{A18.}$$

From A15, $(A-\rho)>0$, thus $\tau_K \geq 0$ requires that $\sigma(A-n) > (A-\rho)$. Algebraicmanipulation shows $\tau_K \leq 1$ as

$$\tau_K = \frac{A\sigma(1-\alpha)(A-\rho)-[(1-\alpha)(A-\rho)(n\sigma+A-\rho)]}{A\sigma(1-\alpha)(A-\rho)+A\sigma^2\alpha(A-n)]} \leq 1$$

APPENDIX 2

Proof of Proposition 2.

The first order conditions are

$$\frac{dH}{dc} = 0 \Rightarrow \alpha c^{\alpha(1-\sigma)-1}(1-\tau_s)^{(1-\alpha)(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} = \theta_1 \quad \text{A2.1.}$$

$$\frac{dH}{d\tau_s} = 0 \Rightarrow N(t) c^{\alpha(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\alpha)(1-\tau_s)^{(1-\alpha)(1-\sigma)-1} = \theta_2 B h \eta \quad \text{A2.2.}$$

$$\frac{\dot{\theta}_1}{\theta_1} = (\rho - A) \quad \text{A2.3}$$

$$\frac{\dot{\theta}_2}{\theta_2} = \rho - B\eta \quad \text{A2.4}$$

From A2.1 we have

$$\frac{\dot{\theta}_1}{\theta_1} = \left\{\alpha(1-\sigma)-1\right\}\gamma_c + (1-\sigma)(1-\alpha)B\eta\tau_s \quad \text{A2.5.}$$

Comparing A2.3 and A2.5

$$\rho - A = -\{1-\alpha(1-\sigma)\}\gamma_c + (1-\sigma)(1-\alpha)B\eta\tau_s$$

$$\text{or } \gamma_c = \frac{(1-\sigma)(1-\alpha)B\eta\tau_s + A - \rho}{\{1-\alpha(1-\sigma)\}} \quad \text{A2.6.}$$

From A2.2 we have

$$\{(1-\alpha)(1-\sigma)-1\}\gamma_h + \alpha(1-\sigma)\gamma_c + n = \frac{\dot{\theta}_2}{\theta_2} \quad \text{A2.7}$$

Comparing A2.4 and A2.7 we have

$$\{(1-\alpha)(1-\sigma)-1\}\gamma_h+\alpha(1-\sigma)\gamma_c+n=\rho-B\eta$$

Substituting the value of $\gamma_h=B\eta\tau_s$ andthat of γ_c from A2.6, we have

$$\{(1-\alpha)(1-\sigma)-1\}\eta\tau_s B+\alpha(1-\sigma)[\frac{(1-\sigma)(1-\alpha)B\eta\tau_s-\rho+A}{\{1-\alpha(1-\sigma)\}}]+n=\rho-B\eta \qquad \text{A2.8}$$

orsolving τ_s in terms of parameters:

$$\tau_s=\frac{\{1-\alpha(1-\sigma)\}(n+B\eta)+A\alpha(1-\sigma)-\rho}{B\eta\sigma} \qquad \text{A2.9.}$$

The optimal tax rate will lie between 0 and 1 if the following condition is satisfied.

$$(n+B\eta)[1-\alpha(1-\sigma)]+A\alpha(1-\sigma)-B\eta\sigma<\rho<\{1-\alpha(1-\sigma)\}(n+B\eta)+A\alpha(1-\sigma)$$

and $\gamma_h=B\eta\tau_s=\dfrac{\alpha(1-\sigma)[A-n-B\eta]+(n+B\eta-\rho)}{\sigma}$ A2.10.

From A2.6

$$\gamma_c=\frac{[(1-\alpha)(1-\sigma)(n+B\eta)-\rho]}{\sigma}+A\frac{[\sigma+\alpha(1-\alpha)(1-\sigma)^2]}{\sigma\{1-\alpha(1-\sigma)\}} \qquad \text{A2.11.}$$

From $\dot{K}=y_c-Nc$

$$\gamma_K=\gamma_c+n \qquad \text{A2.12.}$$

APPENDIX 3

Extended Model when both sectors are taxed:

$$u(c)=\int_0^{\infty}\frac{(c^{\alpha}s^{1-\alpha})^{1-\sigma}-1}{(1-\sigma)}e^{-\rho t}N(t)dt \tag{A3.1}$$

$$y_c = AK \tag{A3.2}$$

$$\mathrm{y_s} = \mathrm{B(Nh)} \tag{A3.3}$$

$$N(t)=N_0e^{nt} \tag{A3.4}$$

$$\dot{h}=\eta\frac{(\phi G)}{N} \tag{A3.5}$$

$$G = T = \tau_K y_c + p_s\tau_s y_s \tag{A3.6}$$

From the market clearing condition

$$s=(1-\tau_s)Bh \tag{A3.7}$$

theinvestment function is asfollows:

$$\dot{K}=(1-\tau_K)y_c+(1-\phi)G-Nc \tag{A3.8}$$

From the consumer's equilibrium condition, the value of p_s is solved:

$$\frac{p_c}{p_s}=\frac{MU_c}{MU_s}=\frac{\alpha c^{\alpha(1-\sigma)-1}s^{(1-\alpha)(1-\sigma)}}{(1-\alpha)c^{\alpha(1-\sigma)}s^{(1-\alpha)(1-\sigma)-1}} \tag{A3.9}$$

It is assumed that the value of

$$p_c = 1$$

Therefore $p_s = \frac{(1-\alpha)}{\alpha} \frac{c}{(1-\tau_s)Bh}$ (A3.10)

The current value Hamiltonian function can be formulated as

$$H = N(t)[\frac{c^{\alpha(1-\sigma)} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\tau_s)^{(1-\alpha)(1-\sigma)} - 1}{(1-\sigma)}] + \theta_1[(1-\tau_K)AK + (1-\phi)\tau_K AK + \tau_s \frac{(1-\alpha)}{\alpha} \frac{(1-\phi)cN}{(1-\tau_s)} - cN] + \theta_2 \eta \frac{\phi}{N}(\tau_K AK + \tau_s \frac{(1-\alpha)}{\alpha} \frac{cN}{(1-\tau_s)}) \quad \text{(A3.11)}$$

The control variables are c, τ_K, τ_s and the state variables are *K*, *h*.

From the first order conditions of the control variables we get

$$\frac{dH}{dc} = 0$$

or $\alpha N c^{\alpha(1-\sigma)-1} B^{(1-\alpha)(1-\sigma)} h^{(1-\alpha)(1-\sigma)} (1-\tau_s)^{(1-\alpha)(1-\sigma)} + \theta_1 \tau_s N \frac{(1-\alpha)}{\alpha} \frac{(1-\phi)}{(1-\tau_s)} + \theta_2 \eta \phi \tau_s \frac{(1-\alpha)}{\alpha(1-\tau_s)} = \theta_1 N$ (A3.12)

$$\frac{dH}{d\tau_K} = 0$$

or $\theta_2 \eta = \theta_1 N$ (A3.13)

Taking the logarithm of both sides and differentiating with respect to time,

$$\frac{\dot{\theta}_1}{\theta_1} + n = \frac{\dot{\theta}_2}{\theta_2} \quad \text{(A3.14)}$$

From $\frac{dH}{d\tau_s} = 0$

andsubstituting the value of θ_2 into the equation from equation (A3.13)

$$\alpha(1-\tau_s)^{(1-\alpha)(1-\sigma)+1} c^{\alpha(1-\sigma)-1} B^{(1-\alpha)(1-\sigma)} h^{((1-\alpha)(1-\sigma)} = \theta_1 \quad \text{(A3.15)}$$

Taking the logarithm of both sides and differentiating with respect to time,

$$\{\alpha(1-\sigma)-1\}\gamma_c + (1-\sigma)(1-\alpha)\gamma_h = -\frac{\dot{\theta}_1}{\theta_1} \tag{A3.16}$$

The co-state equation of the state variable K is

$$\dot{\theta}_1 = \rho\theta_1 - \frac{dH}{dK} \tag{A3.17}$$

Now $\frac{dH}{dK} = \theta_1(1-\tau_K)A + \theta_1(1-\phi)\tau_K A + \theta_2\eta\frac{\phi}{N}\tau_K A$ (A3.18)

Substituting this value from (A3.18) into equation (A3.17)

$$\dot{\theta}_1 = \theta_1(\rho - A)$$

or $\frac{\dot{\theta}_1}{\theta_1} = (\rho - A)$ (A3.19)

The other co-state equation is

$$\dot{\theta}_2 = \rho\theta_2 - \frac{dH}{dh} \tag{A3.20}$$

Now $\frac{dH}{dh} = Nc^{\alpha(1-\sigma)}(1-\tau_s)^{(1-\alpha)(1-\sigma)}B^{(1-\alpha)(1-\sigma)}(1-\alpha)h^{(1-\alpha)(1-\sigma)-1}$

$$\dot{\theta}_2 = \rho\theta_2 - \frac{dH}{dh}$$

or $\frac{\dot{\theta}_2}{\theta_2} = \rho - \frac{N(t)c^{\alpha(1-\sigma)}(1-\tau_s)^{(1-\alpha)(1-\sigma)}B^{(1-\alpha)(1-\sigma)}(1-\alpha)h^{(1-\alpha)(1-\sigma)-1}}{\theta_2}$ (A3.21)

Substituting the value of θ_2 from equation (A3.13) into equation (A3.15)

$$\frac{\dot{\theta}_2}{\theta_2} = \rho - \frac{\eta}{(1-\tau_s)\alpha}\left(\frac{c}{h}\right)(1-\alpha) \tag{A3.21.A}$$

From the human capital accumulation function we get

$$\gamma_h = \frac{\dot{h}}{h} = \eta\phi\tau_K Ak + \frac{\tau_s}{(1-\tau_s)}\frac{(1-\alpha)}{\alpha}\left(\frac{c}{h}\right) \tag{A3.22}$$

Proof of Proposition 5:

From equations (A3.14) and (A3.19) we get

$$\frac{\dot{\theta}_2}{\theta_2} = (\rho - A + n) \tag{A3.23}$$

Equating equation (A3.21.A) with(A3.23) we get

$$(\frac{c}{h}) = \frac{(A-n)(1-\tau_s)\alpha}{\eta(1-\alpha)} \tag{A3.24}$$

From equation (A3.24) it is clear that as $(\frac{c}{h})$ is constant, $\gamma_c = \gamma_h$

From equation (A3.16) and (A3.19) we get

$$\{\alpha(1-\sigma)-1\}\gamma_c + (1-\sigma)(1-\alpha)\gamma_h = (A-\rho)$$

or $\gamma_c[\{1-\alpha(1-\sigma)\} - (1-\alpha)(1-\sigma)] = (A-\rho)$

$$\text{or } \gamma_c = \frac{(A-\rho)}{\sigma} \tag{A3.25}$$

From equation (A3.12) and using equations (A3.13) and equation (A3.15) we obtain

$$\frac{\tau_s}{(1-\tau_s)\alpha} = 0 \text{ implying } \tau_s = 0$$

The growth rate of physical capital is

$$\gamma_K = (1-\tau_K)\frac{y_c}{K} + (1-\phi)\frac{G}{K} - \frac{cN}{K}$$

After simplification we get

$$\gamma_K = A(1-\phi\tau_K) + (\frac{c}{kh}) \tag{A3.26}$$

Where $k = \frac{K}{Nh}$ in the steady state, γ_K is constant so $(\frac{c}{kh})$ should be constant.

We have already found that $\frac{c}{h}$ is constant.Therefore, k is constant.

Hence,

$$\gamma_K = n + \gamma_h = \frac{(A-\rho+n\sigma)}{\sigma} \tag{A3.27}$$

Again from equation (A3.25) we get

$$\gamma_c = \gamma_h = \frac{(A-\rho)}{\sigma}$$

andfrom equation (A3.22)

$$\gamma_h = \eta\phi\tau_K Ak + \frac{\tau_s}{(1-\tau_s)}\frac{(1-\alpha)}{\alpha}(\frac{c}{h})$$

Since $\tau_s = 0$, therefore

$$\frac{(A-\rho)}{\sigma} = \eta\phi\tau_K Ak$$

Hence, $k = \frac{(A-\rho)}{\sigma\eta\phi A\tau_K}$ (A3.28)

Equating the value of γ_K from (A3.26) and (A3.27) and using (A3.24) and (A3.28) we obtain

the value of $\tau_K = \frac{\{(A-\rho)-\sigma(A-n)\}(1-\alpha)(A-\rho)}{\sigma[(1-\phi-A)(1-\alpha)(A-\rho)-(A-n)\alpha\sigma\phi A]}$ (A3.29)