ABSTRACT: The purpose of this paper is to extend Uzawa’s two-sector model of a national economy to an economy with any number of regions and interregional tourism. The paper studies interregional economic development with interactions between wealth accumulation, economic structure, interregional trade and tourism under assumptions of profit maximization, utility maximization, and perfect competition. The model is unique in interregional neoclassical growth theory in that it introduces endogenous tourism within a general equilibrium framework. The model is built on microeconomic foundations. It not only extends the well-known Solow growth model and the Uzawa two-sector model to a national economy with any number of heterogeneous regional economies, but also introduces tourist flows between regions. We demonstrate that the movement of the J-regional economy can be described by J+1 differential equations. We simulate the movement of a national economy with three regions. We show that the dynamic system has a unique equilibrium. We carry out comparative dynamic analysis with regard to the propensity to tour a region, the cost of travel from one region to another, the total factor productivity of a region’s industrial sector, the total factor productivity of a region’s service sector, the propensity to save, the parameters of a region’s amenity, the propensity to consume housing, and the national population. We demonstrate the dynamic effects of these changes on national GDP, wealth, and tourist patterns.

KEY WORDS: multi-region economic dynamics; tourism; regional disparities in wealth and income; wealth accumulation; amenity

JEL CLASSIFICATION: R11, O11, O18, L83

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SPATIAL AGGLOMERATION AND ECONOMIC DEVELOPMENT WITH THE INCLUSION OF INTERREGIONAL TOURISM
1. INTRODUCTION

The main purpose of this study is to introduce tourism into a multi-regional growth model. Although tourism has played an increasingly important role in national economies partly in tandem with rapid economic growth in different parts of the world, there are only a few formal models based on the microeconomic foundation of dynamic interactions with tourism. This study tries to explore dynamic interactions between tourism and economic growth. Although tourism connects different economies, in trade theory it differs from common trade in trade theory in the sense that it converts some non-traded goods into tradable ones. Tourism goods such as monuments of national heritage, historical sites, beaches, and hot springs, are non-tradable, as they are consumed in situ: people from other regions can enjoy the goods only by visiting the spot. Tourism may interact with economic growth in different ways. On the one hand, tourism creates job opportunities and income for regional economies (Sinclair 2002; Lee and Chang 2008; Schubert et al. 2011; Seentanah 2011; Sun 2014). Tourism also attracts resources such as labour and capital from other sectors of the local economy. The income generated by tourism encourages development of other economic activities in the region and may also help other regional economies to develop. Economists have shown an increasing interest in tourism (e.g., Sinclair and Stabler 1997; Hazari and Sgro 2004; Hazari and Lin 2011). Nevertheless, as observed by Chao et al. (2009) the study of tourism has been mainly static. It is necessary to build dynamic models of tourism on microeconomic foundations. Another important issue is the relationship between economic structural changes and tourism. As tourism uses national resources, the development of tourism affects the economic structure (e.g., Corden and Neary 1982; Copeland 1991; Oh, 2005; Zeng and Zhu 2011). In order to fully understand the possible effects of tourism on national economic development and economic structure it is necessary to build a dynamic general equilibrium framework. Dwyer et al. (2004) discuss the need for dynamic general equilibrium modelling when studying tourism and its interaction with the rest of the economy. Blake et al. (2006) also address the issue. Nevertheless, there are few dynamic economic models of the dynamic interdependence between economic growth and tourism with a micro-behavioural foundation. We refer to studies on the Uzawa two-sector model by Uzawa (1963), Galor (1992), Mino (1996), Cremers (2006), Li and Lin (2008), and Stockman (2009).
This study proposes a multi-regional growth model with interregional trade and economic structure. The growth mechanism is based on the Solow growth model and ideas of possible mutual benefits from free trade. Each region's economic structure is based on the Uzawa two-sector growth model. As far as goods trade is concerned, we follow the Oniki-Uzawa model (for instance, Oniki and Uzawa 1965; Frenkel and Razin 1987; Nishimura and Shimomura 2002; Sorger 2002). The Oniki-Uzawa model deals with a two-country world economy with two goods. The Oniki-Uzawa modelling framework is often applied to study dynamic interdependence between trade patterns and economic growth. It has been developed for international economies, omitting population migration between national economies. However, regional agglomeration has become increasingly pronounced in various parts of the world as metropolitan areas attract more people (e.g., Bairoch 1993). Kuznets (1966) attributes spatial agglomeration to industrialization. In fact, the early development theories of Myrdal (1957) and Hirschman (1958) argue for dynamic interactions between industrial growth and the geographical concentration of industry: industrialization attracts resources to a given location and the resulting agglomeration stimulates growth. The contemporary literature on economic geography and economic development formalizes these dynamic processes in models based on different factors of nonlinear dynamics (e.g., Krugman 1991; Fujita et al. 1999; Forslid and Ottaviano 2003; Zhang, 1991, 2016). Nevertheless, most models in economic geography have not succeeded in including capital accumulation and infrastructures formation as endogenous processes of industrialization and agglomeration. This paper concentrates on the study of interregional development with capital accumulation, taking into account factors such as environment and regional economic structure. It should be noted that our approach differs from the so-called new economic geography (e.g., Krugman 1991, 1993; Ottaviano et al. 2002; Pflüger 2004; Charlot 2006; Picard and Tabuchi 2010). In almost all the dynamic models of the new economic geography, physical capital is completely neglected and regional amenities do not play a significant role in determining land rent and population mobility. Although this approach claims to have “enabled researchers to gain further insights into the space economy and its transition” (Tabuchi 2014: 50), it is difficult to imagine any modern economy whose dynamics can be properly modelled with neither wealth nor capital accumulation. Moreover, as Tabuchi (2014: 50) observes, “The scopes of most of the theoretical studies published thus far have been limited to two regions in
order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, it is no doubt that the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur.”

The paper basically follows the neoclassical growth theory, emphasizing the role of wealth accumulation, tourism, and amenity in regional growth and agglomeration. Amenities have increasingly been the focus of spatial scientists (Glaeser et al. 2001; Partridge et al. 2008a, 2008b; Chen et al. 2013). There is a large body of literature on amenities and spatial economics; for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown et al. (1997), life cycle studies by Clark and Hunter (1992), and research on rural development by Deller et al. (2001). Zhang (1993b) first introduced spatial amenity into utility in a general equilibrium framework, and then spatial amenity into a formal regional growth model (Zhang 2016). As Chen et al. (2013) observe, “Empirical evidence demonstrates a growing link between the presence of high-valued natural amenities—including pleasant climate and proximity to lakes, oceans, forests, and mountains—and higher rates of population and income growth in the U.S.” Chen et al. develop a two-region model in which labour distribution, production externalities, and natural resources are endogenous. They also show that a strong preference for natural amenities tends to lead to population dispersion. As Chen (2013: 256) points out, although a “number of empirical studies have demonstrated the positive association between rural growth and natural amenities (e.g., McGranahan, 1999, Deller et al., 2001; Shumway and Otterstron, 2001; Kim et al., 2005; Partridge et al., 2008a, 2008b), none have examined the pattern of relative population distribution across amenity-based areas nor sought to develop a theoretical model of amenity-led migration that explains this distribution.”

After reviewing the literature on the relationship between amenities and spatial economic development, Storper and Scott (2009: 153) state: “… any approach to urban dynamics that is not clearly linked to a basic logic of genesis and early growth must be deficient with regard to its central conceptual capabilities, in the
sense that explanation will now be confined to marginal or intermediate adjustments, as opposed to core processes." This study takes account of the role of amenities by following Zhang (2016).

It is mathematically very difficult to take account of this obvious feature of tourism in modelling economic growth with multiple cities, regions or/and nations. This study succeeds in introducing tourism partly by applying the alternative approach to household behavior proposed by Zhang (1993a). This paper is a synthesis of Zhang’s (2012) growth model with tourism and his multi-regional growth model (2016). Zhang (2016) develops a multi-growth model within Uzawa’s well-known two-sector growth model. The model is based on Zhang’s regional growth models (Zhang, 1996, 2009) and Uzawa’s two-sector growth model. Zhang (2012) proposes a growth model including tourism. This study generalizes the previous papers mainly by synthesizing the main ideas within a compact framework.

This paper is organized as follows. Section 2 defines the multi-region model with wealth accumulation, economic structure, interregional migration condition, and tourism. Section 3 identifies the differential equations that can be used to simulate the model, plots the movement of the model, demonstrates the existence of a unique equilibrium point, and proves the stability of the equilibrium point. Section 4 carries out comparative dynamic analysis with regard to some parameters. Section 5 concludes the study. The main analytical results of Section 3 are proved in the Appendix.

2. A MULTI-REGIONAL GROWTH MODEL WITH TOURISM

This paper is an extension of Zhang’s multi-regional growth model (Zhang 2016) and Zhang’s growth model with tourism (2012). This model is influenced by neoclassical trade theory with capital accumulation (Uzawa 1961; Oniki and Uzawa 1965; Brecher, et al. 2002; Sorger, 2002). Each region produces one good and service. Most aspects of the goods sector in our model are similar to the neoclassical one-sector growth model. There is only one (durable) good in the national economy under consideration. Households own assets of the economy and distribute their income between consuming and saving. Production sectors or firms use capital and labour. Exchanges take place in perfectly competitive
markets. Production sectors sell their product to households or to other sectors and households sell their labour and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Only households save, which implies that all firms’ earnings are distributed in the form of payment to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All household savings are absorbed by firms. We require saving and investment to be equal at any point in time.

The national economy consists of $J$ regions, indexed by $j = 1, \ldots, J$. We assume perfect competition in all the markets both within each region and between the regions. Commodities are traded without any barriers. We neglect transport costs. We measure prices in terms of the commodity and the price of the commodity is unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the $j$th region. The interest rate is equalized throughout the national economy, i.e., $r(t) = r_j(t)$. The population $N$ is homogenous. People are free to choose their residential location and people work and reside in the same region. Each region has fixed land $L_j$, which is homogenous within each region. The assumption of zero transportation cost of commodities implies price equality for the commodity between regions. As amenities and land are immobile, wage rates and land rent may vary between regions. We use subscripts, $i, s$, to denote the industrial and services sectors, respectively. Let $F_{qj}(t)$ stand for the output levels of $q$’s sector in region $j$ at time $t$, $q = i, s$

### 2.1 Industrial sectors

There are two productive factors, capital, $K_{qj}(t)$, and labour, $N_{qj}(t)$, at each point in time $t$. We specify the production functions of the two sectors in each region as follows

$$F_{qj}(t) = A_{qj}K_{qj}^{a_q}(t)N_{qj}^{\beta_q}(t), \quad q = i, s, \quad j = 1, \ldots, J.$$
Labour and capital earn their marginal products, and firms earn zero profits. The interest rate and wage rates are determined by markets. The marginal conditions of the industrial sectors are given by

\[ r(t) + \delta_{kj} = \frac{\alpha_{ij} F_{ij}(t)}{K_{ij}(t)} , \quad w_j(t) = \frac{\beta_{ji} F_{ij}(t)}{N_{ij}(t)} , \]  

where \( \delta_{kj} \) is the depreciation rate of physical capital in region \( j \). It should be noted that our approach to firm location is oversimplified, as there are many other factors which affect the location of firms. For literature on identifying the factors that affect the location choice of firms, see, for instance, Lee and Mansfield (1996), Henisz (2000), Busse and Hefeker (2007), Almazan et al. (2007), De Beule and Duanmu (2012), and Colombo and Dawid (2014).

### 2.2 Service sectors

We use \( p_j(t) \) to stand for region \( j \)'s services price. The marginal conditions for the service sectors are given by

\[ r(t) + \delta_{kj} = \frac{\alpha_{sj} p_j(t) F_{sj}(t)}{K_{sj}(t)} , \quad w_j(t) = \frac{\beta_{sj} p_j(t) F_{sj}(t)}{N_{sj}(t)} . \]  

### 2.3 Behaviour of consumers

In order to define incomes it is necessary to determine land ownership structure. Land properties may be distributed in multiple ways under various institutions. To simplify the model, we accept the assumption of absentee landowner, in the sense that all the land rent is spent outside the economic system. This assumption is made to simplify the dynamic system. Each worker gets income from wealth ownership and wages. Region \( j \)'s land is \( L_j \) and the land rent is \( R_j(t) \).

Consumers decide on choice of lot size, consumption level of services and commodities, and how much to save. This study uses the approach to consumers’ behaviour proposed by Zhang (1993) which makes it possible to solve many important (national) economic problems, such as growth problems with heterogeneous households, which are analytically intractable by the traditional
approaches in economics. Let $\bar{k}_j(t)$ stand for the per capita wealth in region $j$. Each consumer of region $j$ obtains income

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t),$$

from the interest payment and the wage payment. The total value of wealth that a consumer of region $j$ can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is then equal to

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t).$$

The disposable income is used for saving and consumption. It should be noted that the variable, $\bar{k}_j(t)$, in the above equation is considered as a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time $t$ by selling all his wealth. Hence, at time $t$ the consumer has a total amount of income equalling $\hat{y}_j(t)$ to distribute between consuming and saving.

In this study we measure housing by the lot size that the representative household uses to provide the service of housing. At each point in time, the consumer in region $j$ distributes the total available budget between lot size, $l_j(t)$, saving, $s_j(t)$, tourist consumption in region $q$, $c_{jq}(t)$, consumption of goods, $c_{ij}(t)$, and consumption of services, $c_{sj}(t)$. Let $d_{jq}(t)$ stand for the visits to region $q$ by region $j$'s representative household. The total cost for touring regions is

$$\sum_{q, q \neq j} (t_{jq} + p_q(t)c_{jq}(t))d_{jq}(t),$$

where
where \( r_{j}c_{jq}(t) \) is the travel cost involved in visiting region \( q \), and \( t_{jq} \) are parameters. We assume that the travel costs are proportional to the consumption level. Equation (5) implies that the total spent on tourism is the sum of the expenditures on transportation and consumption. The budget constraints are

\[
c_{q}(t) + p_{j}(t)c_{sj}(t) + R_{j}(t)l_{j}(t) + s_{j}(t) + \sum_{q \neq j}(t_{jq} + p_{q}(t))d_{jq}(t) = \hat{y}_{j}(t),
\]

where \( d_{jq}(t) = c_{jq}(t)\bar{d}_{jq}(t) \). We specify utility functions \( U_{j}(t) \) as follows

\[
U_{j}(t) = \theta_{j}(t)l_{j}^{\eta_{0}}(c_{ij}^{\xi_{0}}c_{sj}^{\gamma_{0}}s_{j}^{\lambda_{0}})\prod_{q \neq j}^{J}d_{jq}^{\epsilon_{0q}}(t), \quad \eta_{0}, \xi_{0}, \gamma_{0}, > 0, \quad \epsilon_{0q} \geq 0, \quad (7)
\]

in which \( \eta_{0}, \xi_{0}, \gamma_{0}, \lambda_{0} \) and \( \epsilon_{0q} \) are a typical person’s elasticity of utility with regard to lot size, commodities, services, savings, and touring region \( q \). We call \( \eta_{0}, \xi_{0}, \gamma_{0}, \lambda_{0} \), and \( \epsilon_{0j} \) propensities to consume lot size, goods, and services, to hold wealth (save), and to tour region \( q \), respectively. In (7), \( \theta_{j}(t) \) is called region \( j \)’s amenity level. Amenities are affected by infrastructure, regional culture and climate (e.g., Kanemoto 1980; Diamond and Tolley 1981; Blomquist et al. 1988; Zhang 1993a). We specify \( \theta_{j} \) as follows

\[
\theta_{j}(t) = \bar{\theta}_{j}N_{j}^{d}(t), \quad j = 1, \ldots, J, \quad (8)
\]

where \( \bar{\theta}_{j} \) is a parameter and \( N_{j}(t) \) is region \( j \)’s population. We don’t specify signs of \( d \) as population may have either positive or negative effects on regional attractiveness. According to Chen et al. (2013: 269), “The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households’ migration decisions are internalized.” We will examine the effects of changes in amenity parameters on not only steady state but also transitory processes of the economic system. Maximizing the utility subject to (6) yields
\[ l_j(t)R_j(t) = \eta_j \hat{y}_j(t), \quad c_j(t) = \xi_j \hat{y}_j(t), \quad p_j(t)c_{sj}(t) = \gamma_j \hat{y}_j(t), \quad s_j(t) = \lambda_j \hat{y}_j(t), \quad (t_{jq} + p_q(t))d_{jq}(t) = \varepsilon_{jq} \hat{y}_j(t), \quad (9) \]

where

\[
\eta_j = \eta_0 \rho_j, \quad \xi_j = \xi_0 \rho_j, \quad \gamma_j = \gamma_0 \rho_j, \quad \lambda_j = \lambda_0 \rho_j, \quad \varepsilon_{jq} = \varepsilon_{0q} \rho_j, \quad \rho_j = \frac{1}{\eta_0 + \xi_0 + \gamma_0 + \lambda_0 + \sum_{q,q \neq j} \varepsilon_{0q}}, \quad j = 1, \ldots, J.
\]

The savings behaviour of the approach in this study is similar to those implied by the Keynesian consumption function and permanent income hypotheses, which are empirically more valid than the assumptions in the Solow model with a constant saving rate and the Ramsey model.

### 2.4 Wealth accumulation

According to the definitions of \( s_j(t) \), the wealth accumulation of the representative household in region \( j \) is given by

\[
\hat{k}_j(t) = s_j(t) - \bar{k}_j(t). \quad (10)
\]

The equation implies that the change in wealth is savings minus dissaving.

### 2.5 Equalization of utility levels between regions

As households are assumed to be freely mobile between the regions, the utility level of people should be equal, irrespective of which region they live in, i.e.

\[
U_j(t) = U_q(t), \quad j, q = 1, \ldots, J. \quad (11)
\]

We omit possible migration costs. Changing houses or moving to another region will cost, but taking account of such changes in the model makes it difficult to describe the behaviour of households. Wage equalization between regions is often used as the equilibrium mechanism of population mobility over space. This study
assumes that households obtain the same level of utility in different regions as the equilibrium mechanism of population distribution between regions.

2.6 The demand and supply balance for services

The region’s households and tourists consume the region’s supply of services

\[ c_{sj}(t)N_j(t) + \sum_{q, q \neq j}^J d_{jq}(t)N_q(t) = F_{sj}(t), \quad j = 1, \ldots, J. \]  

(12)

where \( c_{sj}(t)N_j(t) \) is the total demand for services from the region’s households and \( \sum_{q, q \neq j}^J d_{jq}(t)N_q(t) \) is the demand for the region’s services from the tourists.

2.7 Capital is fully used

The total capital stock \( K(t) \) employed by the production sectors is equal to the total wealth owned by the households of all the regions. That is

\[ K(t) = \sum_{j=1}^J K_j(t) = \sum_{j=1}^J \bar{k}_j(t)N_j(t), \]  

(13)

in which \( K_j(t) = K_{ji}(t) + K_{js}(t) \).

2.8 Full employment of labour

The assumption that labour force is fully employed implies

\[ N_{ij}(t) + N_{sj}(t) = N_j(t). \]  

(14)

2.9 The national population balance

\[ \sum_{j=1}^J N_j(t) = N, \quad j = 1, \ldots, J. \]  

(15)
2.10 Land is fully used

The assumption that land is fully employed implies

\[ l_j(t)N_j(t) = L_j, \quad j = 1, \ldots, J. \]  

(16)

We have thus built the model, which explains the endogenous wealth, regional capital, labour distribution, and tourist patterns. All the markets are perfectly competitive and product, capital, and labour are freely mobile. Our dynamic equations are highly dimensional and nonlinear. Many studies have confirmed the existence of nonlinearities in regional growth (Arbia and Paelinck 2003; Ertur and Gallo 2009; Fotopoulos 2012; Azomahou et al. 2011; Basile et al. 2014). Nevertheless, only a few regional dynamic nonlinear models are based on a microeconomic foundation. Our model is built on the well-established neoclassical growth theory with a microeconomic foundation. We now analyse the behaviour of the nonlinear dynamic model.

3. SIMULATING THE MODEL

The dynamic system is complicated. For illustration, the rest of the study simulates the model. In the Appendix we show that the dynamics of the national economy can be expressed as \( J + 1 \) dimensional differential equations. First, we introduce a variable \( z_1(t) \)

\[ z_1(t) \equiv \frac{r(t) + \delta_{k1}}{w_1(t)}. \]

Lemma

The movement of the national economy is given by the following \( J + 1 \) differential equations with \( z_1(t) \) and \( \left( k_j(t) \right) \) as variables

\[ \dot{k}_j(t) = \Phi_j \left( z_1(t), k_j(t) \right), \quad j = 1, \ldots, J, \]

\[ \dot{z}_1(t) = \Phi_0 \left( z_1(t), \left( k_j(t) \right) \right), \]  

(17)
where $\Phi_j$ and $\Phi_0$ are functions of $z_1(t)$ and $(\bar{k}_j(t))$, defined in the Appendix.

For any given positive values of $z_1(t)$ and $(\bar{k}_j(t))$ at any point in time, the other variables are uniquely determined by the following procedure: $r(t)$ by (A2) $\rightarrow$ $w_j(t)$ by (A4) $\rightarrow$ $p_j(t)$ by (A5) $\rightarrow$ $\hat{y}_j(t)$ by (A6) $\rightarrow$ $c_{ji}(t)$, $c_{ji}(t)$, $d_{jq}(t)$, and $s_j(t)$ by (7) $\rightarrow$ $N_i(t)$ by (A11) $\rightarrow$ $N_j(t)$ by (A10) $\rightarrow$ $R_j(t)$ by (A12) $\rightarrow$ $l_j(t) = L_j / N_j(t)$ $\rightarrow$ $N_{sj}(t)$ by (A13) $\rightarrow$ $N_{ij}(t)$ by (A15) $\rightarrow$ $F_{ij}(t)$ by definition $\rightarrow$ $K_{ij}(t)$ by (A1) $\rightarrow$ $K_j(t) = K_{is}(t) + K_{ij}(t) + K(t)$ by (13) $\rightarrow$ $Y_j(t) = F_{ij}(t) + p_j(t)F_{ij}(t) \rightarrow Y(t) = \sum_j Y_j(t)$.

The lemma provides a computational procedure for following the movement of the economic system with any number of regions. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-region economy. We specify parameter values as follows

$$N = 20, \quad \lambda_0 = 0.75, \quad \xi_0 = 0.1, \quad \eta_0 = 0.07, \quad \gamma_0 = 0.07, \quad d = -0.05,$$

$$\begin{pmatrix}
\alpha_{i1} \\
\alpha_{i2} \\
\alpha_{i3}
\end{pmatrix} = \begin{pmatrix}
1.2 \\
1 \\
0.95
\end{pmatrix}, \quad \begin{pmatrix}
\alpha_{s1} \\
\alpha_{s2} \\
\alpha_{s3}
\end{pmatrix} = \begin{pmatrix}
1.1 \\
1 \\
0.95
\end{pmatrix}, \quad \begin{pmatrix}
\alpha_{t1} \\
\alpha_{t2} \\
\alpha_{t3}
\end{pmatrix} = \begin{pmatrix}
0.32 \\
0.31 \\
0.3
\end{pmatrix}, \quad \begin{pmatrix}
\alpha_{l1} \\
\alpha_{l2} \\
\alpha_{l3}
\end{pmatrix} = \begin{pmatrix}
0.33 \\
0.34 \\
0.325
\end{pmatrix}, \quad \begin{pmatrix}
L_1 \\
L_2 \\
L_3
\end{pmatrix} = \begin{pmatrix}
3 \\
4 \\
3
\end{pmatrix}.$$

$$\begin{pmatrix}
\epsilon_{o1} \\
\epsilon_{o2} \\
\epsilon_{o3}
\end{pmatrix} = \begin{pmatrix}
0.007 \\
0.008 \\
0.01
\end{pmatrix}, \quad \begin{pmatrix}
t_{12} \\
t_{13} \\
t_{21}
\end{pmatrix} = \begin{pmatrix}
0.08 \\
0.09 \\
0.1
\end{pmatrix}, \quad \begin{pmatrix}
t_{23} \\
t_{31} \\
t_{32}
\end{pmatrix} = \begin{pmatrix}
0.11 \\
0.12 \\
0.15
\end{pmatrix}, \quad \begin{pmatrix}
\delta_{t1} \\
\delta_{t2} \\
\delta_{t3}
\end{pmatrix} = \begin{pmatrix}
3.8 \\
3.5 \\
4
\end{pmatrix}, \quad \begin{pmatrix}
\delta_{s1} \\
\delta_{s2} \\
\delta_{s3}
\end{pmatrix} = \begin{pmatrix}
0.05 \\
0.05 \\
0.06
\end{pmatrix}.$$

Region 1 has the highest levels of productivity in the two sectors, region 2 has the next highest, and region 3’s levels of productivity are the lowest. We specify values of $\alpha_{jk}$ close to 0.3. With regard to the technological parameters, for the purposes of illustration what are important in our interregional study are their relative values. The presumed productivity differences between the regions are not very large. It can be seen that the specified values of the land sizes, the preference parameters and the population will not affect our main concerns regarding interactions between the regions. The propensity to travel to the three regions
differs. The representative household has the highest propensity to tour region 3, the second highest propensity to tour region 2, and the lowest propensity to tour region 1. We specify the initial conditions as follows

\[ z_1(0) = 0.089, \quad \bar{k}_1(0) = 5.4, \quad \bar{k}_2(0) = 4.1, \quad \bar{k}_3(0) = 4.1. \]

The movement of the system is plotted in Figure 1. The national output changes slightly over time, and national wealth falls over time, until they arrive both at the equilibrium level. The interest rate rises in association with falling wealth. The wage rates in all the regions fall. Region 1’s total output and two sectors’ output levels rise. Region 2’s total output and the industrial sector’s output level rise and the service sector’s output level falls. Region 3’s total output and two sectors’ output levels fall. People migrate from regions 3 and 2 to region 1. The lot sizes in regions 3 and 2 are enhanced and the lot size in region 1 is reduced. The land rents in regions 2 and 3 are lowered and the land rent in region 1 is increased. Region 1 and region 2’s amenity levels are slightly affected, and region 3’s amenities are slightly improved. Service prices change slightly over time. Travel between any regions is reduced. The wealth levels and consumption of goods and services per household in all three regions are reduced.

Figure 1. The Motion of the Economic System
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It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values as follows:

\[
Y = 39.1, \quad K = 99.1, \quad r = 0.077,
\]

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} =
\begin{bmatrix}
35.35 \\
1.90 \\
1.80
\end{bmatrix},
\begin{bmatrix}
N_1 \\
N_2 \\
N_3
\end{bmatrix} =
\begin{bmatrix}
17.43 \\
1.23 \\
1.34
\end{bmatrix},
\begin{bmatrix}
F_{11} \\
F_{22} \\
F_{33}
\end{bmatrix} =
\begin{bmatrix}
26.91 \\
0.53 \\
0.21
\end{bmatrix},
\begin{bmatrix}
F_{11} \\
N_{i1} \\
N_{i1}
\end{bmatrix} =
\begin{bmatrix}
7.87 \\
13.31 \\
13.31
\end{bmatrix},
\begin{bmatrix}
N_{i2} \\
N_{i2} \\
N_{i2}
\end{bmatrix} =
\begin{bmatrix}
1.43 \\
0.35 \\
0.35
\end{bmatrix},
\begin{bmatrix}
N_{i3} \\
N_{i3} \\
N_{i3}
\end{bmatrix} =
\begin{bmatrix}
1.63 \\
0.16 \\
0.16
\end{bmatrix},
\begin{bmatrix}
N_{j1} \\
N_{j2} \\
N_{j3}
\end{bmatrix} =
\begin{bmatrix}
4.11 \\
0.88 \\
1.17
\end{bmatrix},
\begin{bmatrix}
K_{11} \\
K_{12} \\
K_{13}
\end{bmatrix} =
\begin{bmatrix}
67.89 \\
1.29 \\
0.47
\end{bmatrix},
\begin{bmatrix}
K_{11} \\
K_{11} \\
K_{11}
\end{bmatrix} =
\begin{bmatrix}
21.98 \\
3.68 \\
3.76
\end{bmatrix},
\begin{bmatrix}
F_{11} \\
F_{22} \\
F_{33}
\end{bmatrix} =
\begin{bmatrix}
3.07 \\
0.96 \\
0.97
\end{bmatrix},
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix} =
\begin{bmatrix}
1.37 \\
1.03 \\
0.91
\end{bmatrix},
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix} =
\begin{bmatrix}
2.79 \\
0.11 \\
0.14
\end{bmatrix},
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{bmatrix} =
\begin{bmatrix}
3.29 \\
3.46 \\
3.94
\end{bmatrix},
\begin{bmatrix}
\bar{k}_1 \\
\bar{k}_1 \\
\bar{k}_1
\end{bmatrix} =
\begin{bmatrix}
5.14 \\
3.88 \\
3.46
\end{bmatrix},
\begin{bmatrix}
\bar{l}_1 \\
\bar{l}_2 \\
\bar{l}_3
\end{bmatrix} =
\begin{bmatrix}
0.17 \\
3.25 \\
2.24
\end{bmatrix},
\begin{bmatrix}
c_{i1} \\
c_{i2} \\
c_{i3}
\end{bmatrix} =
\begin{bmatrix}
0.69 \\
0.52 \\
0.46
\end{bmatrix},
\begin{bmatrix}
c_{j1} \\
c_{j2} \\
c_{j3}
\end{bmatrix} =
\begin{bmatrix}
0.45 \\
0.38 \\
0.33
\end{bmatrix},
\begin{bmatrix}
d_{12} \\
d_{13} \\
d_{21}
\end{bmatrix} =
\begin{bmatrix}
0.053 \\
0.065 \\
0.031
\end{bmatrix},
\begin{bmatrix}
d_{22} \\
d_{31} \\
d_{32}
\end{bmatrix} =
\begin{bmatrix}
0.048 \\
0.027 \\
0.033
\end{bmatrix}.
\]

It is straightforward to calculate the three eigenvalues as follows

\[
\{-0.1996, -0.1981, -0.1991\}.
\]

The three eigenvalues are real and negative. The unique equilibrium point is locally stable. This result is important as it guarantees the validity of exercising comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the national economy under (18). In this study we assume that preference, population, regional land, and technology are exogenous. Examining how the national and regional economies react to changes in these
parameters is significant. As the lemma gives a computational procedure to calibrate the movement of all the variables, comparative dynamic analysis is straightforward.

4.1 The propensity to tour region 3 increases

We now examine the effects of the following change in the population's propensity to tour region 3: \( \varepsilon_{03} : 0.01 \Rightarrow 0.02 \). The simulation result is plotted in Figure 2. In Figure 2 the solid lines stand for the variable values before the parameter is changed and the dashed lines stand for the variable values after the parameter is changed. We use solid and dashed lines with the same meanings in the rest of the paper.

The immediate effect is that people in regions 1 and 2 travel more to region 3. Tourists from regions 1 and 3 to region 2 are reduced. Tourists from region 3 to regions 1 and 2 are slightly affected. As region 3 becomes more attractive for touring, the region’s service sector produces more and uses more capital and labour inputs. Region 3’s industrial sector produces less and uses less capital and labour inputs. Some households migrate to region 3 from regions 1 and 2. Region 3’s amenities worsen and lot size is reduced; the other two regions’ amenities improve and lot sizes increase. Region 3’s land rent increases and the other two regions’ land rents are reduced. The prices of services in the three regions increase slightly. The representative household in each region consumes less industrial and service products. Region 3’s GDP and wealth per household increase; the other two regions’ wealth levels and GDPs are reduced. The interest rate increases and wage rates in the three regions are reduced. Both the industrial and service sector in regions 1 and 2 produce less. National GDP is reduced.

It should be noted that empirical studies in the literature show an opposing relationship between tourism boom and economic development (for instance, Balaguer and Cantavella-Jorda 2002; Dritsakis 2004; Drubarry 2004; Oh 2005; Kim et al. 2006). Harzri and Sgro (1995) show that an increase in international tourism leads to a positive effect on the long-run economic growth of a small open economy. Our result shows that as people have a stronger propensity to tour a region, the region’s GDP increases. Nevertheless, the change has a stronger impact on the economic structure: the region’s service sector expands and the industrial
sector is reduced. It should also be noted that the study by Chao et al. (2006) shows that an expansion of tourism can result in capital decumulation in a two-sector dynamic model with a capital-generating externality. Our simulation demonstrates that region 3 employs more capital as the net consequence of the service sector employing more capital and the industrial sector less capital.

Figure 2. The Propensity to Tour Region 3 Increases

4.2 The travel cost from region 1 to region 3 is reduced

We now allow the travel cost from region 1 to region 3 to be reduced as follows: $t_{13} : 0.09 \Rightarrow 0.05$. The simulation result is plotted in Figure 3. We see that people in region 1 travel more to region 3 and tourists from region 1 to region 2 are slightly reduced. The tourists between the other regions are slightly affected. Region’s service sector produces more and uses more capital and labour inputs. Region 3’s industrial sector produces less and uses less capital and labour inputs. There are slight changes in regional population distribution. From Figure 2 we see that the travel cost change has weak effects on most of the variables in the dynamic system.
4.3 A rise in the total factor productivity of region 2’s industrial sector

We now examine the effects of a technological improvement in region 2’s industrial sector as follows: \( A_{i_2} : 1 \rightarrow 1.05 \). The simulation results are plotted in Figure 4. It should be noted that the value of \( A_{i_2} \) after the change is still lower than the value of \( A_{i_1} \). This implies that if all the other conditions are the same, then migration from region 1 to region 2 may reduce national output. The counterintuitive consequence for national output due to technological improvement in a technologically less-advanced region is confirmed in Figure 4. National GDP is reduced when region 2’s productivity improves. Region 2’s technological progress attracts more people to the region from the other two regions. It should be noted that region 1’s total factor productivity is higher than region 2’s and region 2’s is higher than region 3’s. As more people migrate to region 2 from region 1 than from region 3, we see that the net impact on the national income due to the redistribution of labour force is negative. The output levels and two inputs of region 2’s industrial and service sectors increase, and the output levels and two inputs of region 1’s and 3’s industrial and service sectors reduce. Region 2’s GDP and total capital stocks employed by the region are
increased, and region 1’s and 3’s GDP and total capital stocks employed by the two regions are reduced. Region 2’s lot size falls and land rent rises; the other two regions’ lot sizes are increased and land rents are reduced. The interest rate is reduced slightly in tandem with a slight rise in the national physical capital. Region 2’s service prices, wage rate, consumption levels of industrial goods and services, and wealth per capita increased, while these variables are slightly affected in the other two regions. Region 2’s amenities deteriorate, and the other two regions’ amenities improve. The people from region 2 travel more to the other two regions, and people from the other two regions travel less. Travels between regions 1 and 3 are slightly affected.

Another insight we obtain from this analysis concerns the dynamics of wage disparities over time between regions. Wage disparities are caused by many factors, such as spatial differences in education opportunities, knowledge diffusion, the skill composition of the workforce, local interactions, discrimination, and non-human endowments (see, for instance, Glaeser and Maré 2001; Duranton and Monastiriotis 2002; Combes, et al. 2003; Rey and Janikas 2005). From our simulation results we see that wage disparity is strongly affected by changes in technology. This also hints that if technological differences between regions are not large, wage rates may converge if the other factors weakly affect the differences. It should be remarked that there are different studies on regional economic growth with endogenous knowledge (Florida et al. 2008; Brunow and Hirte 2009; Banerjee and Jarmuzek 2010; Fleisher, et al. 2010; Batabyal and Nijkamp 2013). Although our study does not include endogenous technological change, the literature on regional economic growth and knowledge should enable us to further generalize our modelling.
4.4 The total factor productivity of region 3’s service sector is enhanced

We now examine the effects of the following technological improvement in region 3’s service sector: \( A_{s3} : 0.95 \rightarrow 1 \). The simulation results are plotted in Figure 5. As in the previous case where region 2’s total factor productivity was enhanced, national output and wealth are reduced by the technological improvement. This happens as people migrate from the advanced region to a less technologically advanced region, leading to reduction of the national income and wealth. Region 2’s lot size falls slightly and land rent rises; the other two regions’ lot sizes increase slightly and land rents are reduced slightly. The interest rate and wage rates are slightly affected. Service prices in region 3 are reduced. The falling prices attract more tourists from regions 1 and 2. Travels between any other two regions is slightly affected. Region 2 attracts more capital while the other two regions employ less capital. The output levels of Region 2’s two sectors are enhanced. The output levels of the industrial sectors in the other two regions decline. Wealth levels are slightly affected.
4.5 A rise in the propensity to save

Different economic theories predict different effects of changes in the propensity to save. Keynesian economic theory suggests a negative effect of saving on economic growth, while neoclassical growth theory predicts the opposite effect. As only a few growth models with space take account of endogenous savings, regional growth theory has not much to say on how a change in the propensity to save can affect spatial agglomeration and regional economic growth. We now allow the propensity to save to be changed as follows: \( \lambda_0 : 0.75 \Rightarrow 0.77 \). The simulation results are plotted in Figure 6. National output and wealth increase. The interest rate is lowered in tandem with rises in all three regions’ wage rates. The population is redistributed between regions. Region 1 attracts more people from regions 2 and 3. Region 1’s land rent increases and the other two regions’ land rents are reduced. Region 1’s amenities slightly worsen, and the other two regions’ amenities improve. Region 1’s GDP increases and the other two regions’ GDP is reduced. The representative household’s wealth increases in all the regions. Travel between any two regions increases because the households’ disposable income increases. The consumption levels per household of industrial goods and services in the three regions increase.
4.6 Region 3’s amenity parameter is enhanced

We analyse the effects of the following rise in region 3’s amenity parameter: \( \bar{\theta}_3 : 4 \Rightarrow 4.1 \). A rise in this parameter implies that with all the economic conditions fixed region 3 becomes more attractive for people to live in. The simulation results are plotted in Figure 7. All three regions’ amenities improve. National output and wealth fall, as people prefer to live in the less-advanced region. The interest rate rises. In all the regions wage rates fall. Travel between any two regions is slightly affected. Consumption levels and prices are affected but all very slightly. It should be noted that our approach to regional housing markets is related to hedonic price modelling (e.g., Rosen 1974; Helbich et al. 2014). The approach is based on Lancaster’s idea that it is a good’s characteristics that creates utility. When we apply this idea to the housing market, which is tied to environment and land, it implies that environment should affect housing prices (Dubin 1992; Can and Megbolugbe 1997; Sheppard 1997; Malpezzi 2003; McMillen 2010; Ahlfeldt 2011). Our model shows how rent levels are closely related to different regional characteristics.
4.7 A rise in the propensity to consume housing

We now study the effects of the following rise in the population’s propensity to consume housing: \( \eta_0 : 0.07 \Rightarrow 0.08 \). The simulation results are plotted in Figure 8. National output and wealth fall as people devote more of their disposable income to housing. The interest rate rises and wage rates in all the regions fall. The prices of services in the three regions are slightly enhanced. Region 1’s amenities improve slightly and the other two regions’ amenities deteriorate. Land rents in all three regions increase. Some people from region 1 migrate to regions 2 and 3. The region’s lot size increases and the other two regions’ lot sizes are reduced. Region 1’s GDP, the output of the two sectors, and capital employed by the region are all reduced, while the corresponding variables in the other two regions are all enhanced. Consumption and wealth levels per household are reduced in all the regions. Travel between any two regions is reduced.
4.8 A rise in the national population

We now study the effects of the following rise in the nation population: \( N : 20 \rightarrow 21 \). The simulation results are plotted in Figure 9. From Figure 9 we see that all the national and regional aggregated variables increase. Land rents increase and lot sizes are reduced. The microeconomic variables are slightly affected. The representative household from any region changes slightly their travels to any other region.
5. CONCLUSIONS

This paper proposes a multi-regional growth model that includes tourism. The study demonstrates the dynamic interactions between economic growth, economic structural change, interregional trade and tourist flows. The model is unique in this type of interregional neoclassical growth models in that it introduces endogenous tourism within a general equilibrium framework. The model is built on microeconomic foundations. It not only extends both the well-known Solow growth model and the Uzawa two-sector model to a multi-regional economy, but also introduces tourist flows between regional economies for an economy with any number of regions. We demonstrate that the motion of the \( J \)-regional economy can be described by \( J + 1 \) differential equations. We also simulate the global economy with three regions. We show that the dynamic system has a unique equilibrium. We carry out comparative dynamic analysis with regard to the propensity to tour region 3, the travel cost from region 1 to region 3, the total factor productivity of region 2’s industrial sector, the total factor productivity of region 3’s service sector, the propensity to save, region 3’s amenity parameter, the propensity to consume housing, and the national population. We demonstrate the

Figure 9. A Rise in the National Population

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dynamic effects of these changes on national GDP, wealth, and tourist patterns. It should be remarked that as the paper is an extension and generalization of the well-known Uzawa two-sector growth model, we could further generalize the model in multiple directions. As the model is structurally general, it is possible to deal with various national and regional growth issues. It is straightforward to analyse the model’s behaviour with other forms of production or utility functions. Households should be heterogeneous. Also, much attention has been paid in economic geography to issues related to tax competition between regions (for instance, Andersson and Forslid 2003; Baldwin and Krugman 2004; Bayindir-Upmann and Ziad 2005; Borck and Pflüger 2006). The dynamic equilibrium framework can be extended to examine these issues.

**APPENDIX: PROVING THE LEMMA**

We now demonstrate a procedure to determine the dynamics of the system in two differential equations. First, from (1) we obtain

\[ z_j = \frac{r + \delta_k}{w_j} = \frac{a_j N_{ij}}{K_{ij}} = \frac{b_j N_{sj}}{K_{sj}}, \]  

(A1)

where

\[ a_j = \frac{\alpha_{ij}}{\beta_{ij}}, \quad b_j = \frac{\alpha_{sj}}{\beta_{sj}}. \]

Insert \( z_j / a_j = N_{ij} / K_{ij} \) in \( r + \delta_{kj} = \alpha_{ij} F_{ij} / K_{ij} \) from (1)

\[ r(z_j) = \frac{\alpha_{ij} A_{ij} z_j^{b_j}}{a_j^{b_j}} - \delta_{kj}, \quad j = 1, ..., J. \]  

(A2)

From (A2) we get
From (A1) and (A2) we have

\[ w_j(z_1) = \frac{r + \delta_k}{z_j}. \quad (A4) \]

From \( z_j = b_j N_{sj} / K_{sj} \) and (1) we have

\[ p_j(z_1) = \frac{b_{sj}^\beta (r + \delta_k)}{\alpha_{sj} A_{sj} z_j^{\beta_{sj}}}. \quad (A5) \]

From (12) and (9) we have

\[ \gamma_j \hat{y}_j N_j + \sum_{q, q \neq j}^J \left( \frac{\varepsilon_{aq} p_j}{t_{aq} + p_j} \right) \hat{y}_q N_q = p_j F_{sj}, \quad j = 1, \ldots, J. \quad (A6) \]

Insert (1) in (A6)

\[ \gamma_j \hat{y}_j N_j + \sum_{q, q \neq j}^J \left( \frac{p_j}{t_{aq} + p_j} \right) \hat{y}_q \varepsilon_{aq} N_q = \frac{w_j N_{sj}}{\beta_{sj}}, \quad j = 1, \ldots, J. \quad (A7) \]

By (3) we have

\[ \hat{y}_j(z_1, \bar{k}_j, \bar{r}) = (1 + r)\bar{k}_j + w_j. \quad (A8) \]

Substitute \( l_j = L_j / N_j \), (8), and (9) into (7)
\[ U_j = \frac{\bar{\theta}_j t_j N_j^d}{\bar{\theta}_j t_j N_j} \frac{\xi_j^{\gamma_0} \gamma_j^{\nu_0} \lambda_j^{\nu_0}}{p_j^{\nu_0}} \hat{y}_j^{\nu_j}, \quad (A9) \]

where

\[ t_j(z_1) = \prod_{q,q \neq j} \left( \frac{\xi_j \gamma_j}{t_j + p_q} \right)^{\nu_0 q}, \quad \omega_j = \xi_0 + \gamma_0 + \lambda_0 + \sum_{q,q \neq j} \nu_0 q. \]

Apply \( U_j = U_q \) to (A9)

\[ N_j = \Lambda_j N_1, \quad (A10) \]

where

\[ \Lambda_j(z_1, \{k_q\}) = \left( \frac{\bar{\theta}_j \xi_j^{\gamma_0} \gamma_j^{\nu_0} \lambda_j^{\nu_0}}{\bar{\theta}_j t_j N_j^{d-\nu_0}} \frac{1}{(d-\nu_0)} \left( \frac{\hat{y}_j^{\nu_j}}{\hat{y}_j^{\nu_j}} \right) \right)^{1/(d-\nu_0)}. \]

Insert (A10) in (13)

\[ N_1(z_1, \{k_q\}) = \frac{N_j}{\sum_{j=1}^{J} \Lambda_j}, \quad \Lambda_j = 1. \quad (A11) \]

With (A10) and (A11) we determine the population distributions as functions of \( z_1, \{k_q\}, \) and \( \bar{r} \). By \( l_j R_j = \eta_j \hat{y}_j \) and \( l_j N_j = L_j \), we have

\[ R_j(z_1, \{k_q\}) = \frac{\eta_j \hat{y}_j N_j}{L_j}. \quad (A12) \]

Insert (A8) in (A7)
\[ N_{sj}(z_j, (\vec{k}_q)) = \left( \frac{(1 + r)\vec{k}_j}{w_j} + 1 \right) \beta_{sj} \gamma_j N_j + \frac{\beta_{sj} p_j}{w_j} \sum_{q, q \neq j} \left( \frac{(1 + r)\vec{k}_q + w_q}{t_{qj} + p_j} \right) \varepsilon_{qj} N_q. \] (A13)

From \( N_{ji} + N_{js} = N_j \) and (A10), we have

\[ N_{ij}(z_1, (\vec{k}_q)) = N_j - N_{sj}, \quad j = 1, \ldots, J. \] (A14)

From equation (13), we have

\[ \sum_{j=1}^{J} (K_{ij} + K_{sj}) = \sum_{j=1}^{J} \vec{k}_j N_j. \] (A15)

Insert (A1) in (A15)

\[ \sum_{j=1}^{J} \left( \frac{a_j N_{ij} + b_j N_{sj}}{z_j} \right) = \sum_{j=1}^{J} \vec{k}_j N_j. \] (A16)

Insert (A14) in (A16)

\[ \bar{a} = \sum_{j=1}^{J} \bar{a}_j N_{sj}, \] (A17)

where

\[ \bar{a}(z_1, (\vec{k}_q)) = \sum_{j=1}^{J} \left( \frac{a_j}{z_j} - \vec{k}_j \right) N_j, \quad \bar{a}_j(z_1) = \frac{(a_j - b_j)}{z_j}. \]

Insert (A13) in (A17)

\[ \Phi(z_1, (\vec{k}_q)) = \bar{a} - \sum_{j=1}^{J} \bar{a}_j N_{sj} = 0. \] (A18)
Substitute $s_j = \lambda_j \hat{y}_j$ and $r \bar{k}_j + w_j$ into (9)

$$\dot{k}_j = \Phi_j(z_1, \bar{k}_q) \equiv (1 + r)\lambda_j \bar{k}_j + \lambda_j w_j - \bar{k}_j.$$  \hspace{1cm} (A19)

Taking derivatives of equation (A18) with respect to $t$ yields

$$\dot{z}_1 = \Phi_0(z_1, \bar{k}_q) \equiv -\left( \sum_{j=1}^{J} \Phi_j \frac{\partial \Phi}{\partial \bar{k}_j} \right) \left( \frac{\partial \Phi}{\partial z_1} \right)^{-1},$$  \hspace{1cm} (A20)

where we use (A19). We don’t give expressions as they are tedious. Following the procedure in the lemma we describe the dynamics of the whole system.

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