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TESTING FOR LONG MEMORY IN VOLATILITY IN THE INDIAN FOREX MARKET

ABSTRACT: *This article attempts to verify the presence of long memory in volatility in the Indian foreign exchange market using daily bilateral returns of the Indian Rupee against the US dollar from 17/02/1994 to 08/11/2013. In the first part of the analysis the presence of long-term dependence is confirmed in the return series as well as in two measures of unconditional volatility (absolute returns and squared returns) by employing three measures of long memory. Next, the presence of long memory in conditional volatility is tested using ARMA-FIGARCH and ARMA-FIAPARCH models under various distributional assumptions. The results confirm the presence of long memory in conditional variance for two models. In the last part, the presence of long memory in conditional mean and conditional variance is verified using ARFIMA-FIGARCH and ARFIMA-FIAPARCH models. It is also found thatlong-memory models fare well compared to short-memory modelsin sample forecast performance.*

KEY WORDS: *Long memory, Volatility, India, Forex, fractionally integrated models, FIGARCH, FIAPARCH.*

JEL CLASSIFICATION: G14, G15, F31

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1. INTRODUCTION

Long memory in a time series depicts the persistent temporal dependence present in the data. Presence of long memory in a time series implies that shock at one point in time does not die down quickly. It continues in a decaying fashion, which affects future outcomes. Such series are characterized by distinct but non-periodic cyclical patterns.

The presence of long memory in financial time series data has been an important subject of both theoretical and empirical studies. Since the data points are not independent over time, realizations from the remote past could influence future values.

Bachelier (1900) was the first to propose the theory of random walk to characterize the change of security prices through time. Fama (1965) illustrated that empirical evidence seems to confirm the random walk hypothesis. That is, a series of price changes has no memory. The main theoretical explanation that lies behind this observation is the Efficient Market Hypothesis (EMH), according to which an efficient capital market is one in which security prices adjust rapidly to the arrival of new information, and therefore the current price of securities reflects all information about security. Although the price adjustment may be imperfect, it is unbiased. This implies that even if the market over-adjusts or under-adjusts, an investor cannot predict which will take place at any given time.

Hence, if a statistically significant temporal dependence structure exists within the time series of financial security prices, market participants will immediately exploit them. Financial asset price changes can therefore only be explained by the arrival of new information, which, by definition, cannot be forecast.

If there is a long memory component present in the financial market, it cannot be adequately explained by systems that work with short memory parameters. The short memory property describes the low-order correlation structure of a series and, for short memory, correlations among observations at long lags become negligible. Long memory invalidates standard models based on Brownian motion and martingale assumptions.

There is a lot of literature on volatility modelling. The first type was an unconditional volatility model that assumes volatility to be constant. Later it was found that volatility evolves overtime and shocks persist for a long time. Hence, conditional volatility models such as Autoregressive Conditional

Heteroscedasticity (ARCH) and generalized ARCH (or GARCH) were proposed by Engle (1982) and Bollerslev (1986) respectively. However, these models do not account for long memory in volatility.

Granger and Joyeux (1980) and Hosking (1981) introduced a model of fractional difference in the mean process which is known as autoregressive fractionally integrated moving average (ARFIMA). Baillie et al.(1996) proposed a fractionally integrated GARCH (or FIGARCH) model which introduces a fractional difference operator into the conditional variance equation. The presence of long memory in conditional variance covers the true dependence structure (Mendes and Kolev2006) and perfect arbitrage is not possible when there is long memory present in the returns (Mandelbrot1971). Further, the derivative pricing models based on Brownian motion and the martingale process also become inappropriate in the presence of long-range dependence. Hence, presence of long memory in volatility has important theoretical and practical implications.

Volatility in the foreign exchange market of a country is vitally important to the functioning of the economy. The changes in exchange rate can affect both the price level within the country and external trade. Hence, analysing the dynamics of volatility in the Forex market is of academic as well as practical importance. The presence of long memory could help investors to predict the movement in returns as well as in volatility measures, and in turn make profitable decisions.

The Indian foreign exchange market has witnessed a number of changes in the past decades. From 1947-1971 the Indian rupee followed a par-value system of exchange rate. Following the breakdown of the Bretton Woods system in 1971, the rupee was first pegged to the pound sterling in December1971, and from September 1975 to a basket of currencies. In 1978 the banks in India were permitted to carry out intra-day trading on the foreign exchange market. After the economic reforms of the 1990s, a two-step adjustment of exchange rate in July 1991 effectively ended the fixed rate regime. Following the advice of the Rangarajan committee, the liberalized exchange rate management system (LERMS) was introduced in March 1992, initially having a dual exchange rate system. LERMS was implemented as a transition mechanism and a downward adjustment in exchange rates took place in December 1992. The final convergence of dual exchange rates was made effective from March 1993, signalling the beginning of a market-determined exchange rate regime.

In the recent past India has observed increased fluctuation in its foreign exchange market, especially in the bilateral rate of Indian rupee against US dollar. Therefore, this study attempts to analyse the possible presence of long memory in foreign exchange market volatility in India, taking the Rupee-Dollar exchange rate as a proxy for market activity.

2. LITERATURE REVIEW

To the best of the author's knowledge, there are no studies of long memory in volatility on the Indian Forex market. There are a number of studies that analysed the presence of long memory in rupee returns and we present them below.

Golaknath and Reddy (2002) employed Hurst(R/S) statistics and a variance ratio (VR) test to analyse the effect of long memory on the Indian foreign exchange market. While the VR test did not provide conclusive evidence of the presence of long memory, the R/S statistic indicated the presence of long memory with noise.

Soofi et al. (2008) employed plug-in and whittle methods based on spectral regression analysis to test for long memory in 12 Asian currency daily exchange rates v/s USD. The results showed that, except for the Chinese renminbi, the other 11 exhibited long memory characteristics.

Hseih and Shyu (2009) investigated the long-term dependency behaviour of Asian foreign exchange markets by using rescaled range analysis. Emerging markets in Korea, Taiwan, India, and Thailand showed evidence of long memory in the exchange rate return series, while exchange rate return persistence was not found in the markets of Japan, Australia, Hong Kong, and Singapore. Their results imply that the return-generating processes and presence of long memory depend on the degree of market development.

Senet al. (2010) examined the presence of long memory in nine selected currencies around the globe in terms of the Indian Rupee. The study employed the R/S statistic, modified R/S statistic, Whittle test, and Hurst exponent. They found significant presence of long memory in appreciation and/or depreciation in these nine exchange rates.

Sasikumar (2011) analysed the presence of long memory in Indian Forex market using a number of tests, namely the Hurst exponent, Hurst-Mandelbrot R/S statistic, Lo's R/S statistic, Robinson's semi parametric estimator, and Andrew-Guggenburger modified GPH estimator. The results showed that the Indian Forex market exhibits long memory.

From the literature review it is evident that there is a serious dearth of research in the area of long memory in volatility in the Indian Forex market. The present study places itself in that context. The rest of this article is organized as follows. Section 3 describes data and methodology while section 4 deals with the analysis of the estimated results. Section 5 presents the concluding remarks.

3. DATA AND METHODOLOGY

We have taken the daily bilateral returns of the Indian rupee against the US Dollar from 17/02/1994 to 08/11/2013 for the purpose of analysis. As a proxy for unconditional volatility, we calculate absolute returns and squared returns.

The methodology consists of three parts. In the first part we test for presence of long memory in the return series as well as for unconditional volatility in two measures. For this purpose we employ Mandelbrot's (R/S) statistics test, Lo's modified (R/S) statistics test, and Robinson & Henry's Gaussian Semi-Parametric Estimate test. Next, we test for long memory in conditional volatility using the GARCH framework. We employ two models, ARFIMA-FIGARCH and ARFIMA-APARCH. In the third part we test for long memory both in conditional mean and variance using the GARCH models mentioned in the previous step. The following paragraphs give a brief description of the tests employed.

3.1. (R/S) Statistics by Mandelbrot (1972)

Mandelbrot (1972) proposed a statistic to measure the degree of long-term dependency, in particular, "non periodic cycles". To construct this statistic, consider a time series $X_1, X_2, \ldots\ldots X_n$ with sample mean X and σ_n as the standard deviation. Then the (R/S) statistic Q_n is given by

$$Q_n = \frac{1}{\sigma_n}\left[MAX\sum(X_j - X) - MIN\sum(X_j - X)\right] \quad (1)$$

Mandelbrot suggested a Maximum Likelihood (ML) estimator for σ_n.

3.2. Modified (R/S) Statistics by Lo (1991)

Lo (1991) showed that Mandelbrot's (1972) R/S statistic may be significantly biased when there is short-term dependence in the form of heteroskedasticity or autocorrelation, and suggested the use of a modified rescaled range statistic. The difference between the traditional rescaled range and Lo's modified statistic is the denominator. The modified rescaled range statistic is:

$$Q_{n,q} = \frac{1}{\hat{\sigma}_n}(q)\left[MAX\sum(X_j - X) - MIN\sum(X_j - X)\right] \tag{2}$$

where $= \hat{\sigma}_n(q) = \sigma_n^2 + 2\sum_{j=1}^{q} w_j(q)\tilde{\gamma}_j$

and $w_j(q) = (1\text{-}j)/(q+1)$ and $\tilde{\gamma}_J = 1/n\left[\sum_{i=j+1}^{n}(X_i - X)(X_{i-j} - X)\right]$ for $q<n$.

Lo (1991) provides the assumptions and technical details to allow the asymptotic distribution of $Q_{n,q}$ to be obtained.

3.3. Semiparametric Estimate by (Robinson &Henry1999)

For a time series Xt: X_1,X_2,……..X_n we semi-parametrically model long memory by $f(\lambda)\sim G|\lambda|^{1-2H}$ where $1/2 < H < 1$ and $0 < G < 1$, $f(\lambda)$ being the spectral density of $X_{t.}$ $f(\lambda)$ has a pole at $\lambda = 0$ for $1/2 < H < 1$ (when there is long memory in X_t), $f(\lambda)$ is positive and for $H = 1/2$ (which we identify with short memoryin X_t) and $f(\lambda) = 0$ for $0 < H < 1/2$ (which we describe as negative dependenceor anti-persistence).

The periodogram is defined as

$$I(\lambda) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} x_t \in^{it\lambda}\right|^2 \tag{3}$$

and H is estimated by

$$\widehat{H} = argmind_1 \leq h \leq d_2R(h) \tag{4}$$

where $0<d_1< d_2<1$ and

$$R(h) = \log\left\{\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{1-2h}}\right\} - (2h-1)\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$$

in which $m\in(0, n/2)$ and $\lambda_j = \dfrac{2\pi j}{n}$.

As explained in Robinson (1995), for m = [n/2], $\widehat{H}$ is a form of Gaussian or Whittle estimate under the parametric model $f(\lambda)=G|\lambda|^{1-2H}$ for all $\lambda\in(-\pi, \pi)$ and its asymptotic properties are approximately covered by Fox and Taqqu (1986) (Giraitis and Surgailis1990). Under Gaussian distribution assumption, it is assumed that x_t is linear with independent and identically distributed innovations.

3.4. ARFIMA Model

The ARFIMA model was developed by Granger and Joyeux(1980) and Hosking(1981). The model considers the fractionally integrated process I(*d*) in the conditional mean. The ARFIMA (*p*, ξ, q) for time series process y_t can be expressed as follows:

$$Ø(L)(1-L)^{\xi} y_t = \Theta(L)\varepsilon_t \quad (5)$$

$$\varepsilon_t = z_t \sigma_t, z_t \sim N(0,1) \quad (6)$$

where ξ is the fractional difference parameter, L is a lag operator, ø(L), and Θ(L) are polynomials in the lag operator of orders p and q, respectively, and ε_t is independently and identically distributed with variance σ^2 . Long memory arises through the fractional differencing parameter, ξ , which is allowed to assume any real value. Following Hosking (1981), when ξ ϵ(−0.5, 0.5) the y_t process is stationary and invertible. For such processes, the effect of shocks to ε_t on y_t decays at a slow rate to zero. When ξ = 0, the process is stationary. When ξ ϵ (0, 0.5) the autocorrelations are positive and decay hyperbolically to zero, implying long memory. When ξ ϵ(−0.5, 0) the process is identified as having intermediate memory, since autocorrelations are always negative. However, for ξ = 1, the series follows a unit root process.

3.5. FIGARCH Model

Baillie et al. (1996) introduced long memory in the conditional variance of a GARCH model and proposed the fractionally integrated GARCH or FIGARCH (p,d,q) model, where the conditional variance can be expressed as follows:

$$Ø(L)(1-L)^{d} \varepsilon_t^2 = \omega + [\,1 - \beta(L)]\, v_t \quad (7)$$

where $v_t \equiv \varepsilon_t - \sigma_t^2$. The v_t process can be interpreted as the innovation for the conditional variance and has zero mean serially uncorrelated. To ensure covariance stationarity the roots of Ø(L)and [1 – β(L)] are constrained to lie outside the unit circle. The FIGARCH model offers greater flexibility for modelling the conditional variance. The FIGARCH model in Eq. (7) reduces to a GARCH model when d = 0 and to an IGARCH model when d = 1. The FIGARCH (p,d,q) model imposes an ARFIMA structure on ε_t^2.

The FIGARCH model in Eq. (7) can be rewritten as follows:

$$[1-\beta(L)]\sigma_t^2 = \omega + [1-\beta(L) - ø(L)(1-L)^d]\varepsilon_t^2$$

or, equivalently,

$$\sigma_t^2 = \frac{\omega}{[1-\beta(1)]} + \lambda(L)\ \varepsilon_t^2$$

where

$$\lambda (L) = 1- \frac{\varnothing(L)}{[1-\beta(1)]}(1- L)^d$$

Baillie et al. (1996) state that the impact of a shock on the conditional variance of the FIGARCH (p,d,q) processes decreases at a hyperbolic rate when $0 \leq d <1$. Hence, the long-term dynamics of the volatility are taken into account by the fractional integration parameter d, and the short-term dynamics are modelled through the traditional GARCH parameters.

3.6. FIAPARCH model

The conditional variance equation of the fractionally integrated asymmetric power ARCH (FIAPARCH) (p,d,q) model of (Tsey ,1998) can be written as

$$\sigma_t^2 = \omega + \left\{1-[1-\beta(L)]^{-1}\varphi(L)(1-L)^d\right\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (8)$$

where ω is the ARCH parameter, β is the GARCH parameter, d is the long memory parameter, γ is the asymmetry parameter, and δ is the power term.

4. ANALYSIS

As the first step, we estimate long memory measures for the return series and the unconditional volatility measures. The results are given in Table 1.

Table 1. Long memory estimates for the return series and unconditional volatility measures

Statistic		Returns	Abs. Returns	Sq. Returns
Mandelbrot (R/S) statistic Q		1.428	12.500*	6.3437*
Lo's Modified(R/S) (for q=1,2,5)	$Q_{n,1}$	1.475	10.681`*	5.652*
	$Q_{n,2}$	1.510	9.604*	5.254*
	$Q_{n,5}$	1.490	7.780*	4.561*
Robinson's GSP Estimator	N= T/2	-0.0184 (0.102) (0.064)	0.2769 (0.000) (0.000)	0.2102 (0.000)
	N= T/4	0.058 (0.000)	0.326 (0.000)	0.227 (0.000)
	N= T/8	0.062 (0.000)	0.395 (0.000)	0.262 (0.000)

Note: * indicates significance at 1 % level. p values are in parenthesis

The two R/S statistics for the return series are shown to be statistically insignificant, while significant for the two unconditional volatility measures. However, the GSP estimate confirms the presence of long memory in the return series as well as the unconditional volatility measures. Here, we confirm the presence of long memory in the returns and volatility series. It is to be noted that the extent of long memory is more in the unconditional volatility measures as compared to the return series.

The value of d falls in the range of $0<d<0.5$ for the return series, thus implying a statistically self-similar structure within the analysed time series. For a weak-form informationally efficient market the return series will follow a geometrical Brownian motion, or random walk. The value of d for such a series will be 0. Here, the value of d is statistically significant and greater than zero. Hence it could be inferred that the Indian Forex market is not weak-form informationally efficient. This result is in line with previous studies related to weak-form market efficiency in the Indian Forex market such as Patra (2011) and Sasikumar(2011).

Before we begin testing for long memory in conditional volatility, diagnostic tests such as the Ljung-Box Q stat test and the ARCH-LM test are performed on the returns series. The results are given in Tables 2 and 3.

Table 2. Result of ARCH LM Test

ARCH test	Statistic
F(2,4946)	209.37 (0.0000)
F(5,4940)	105.50 (0.0000)
F(10,4930)	56.535 (0.0000)

Note: p values are in parenthesis

Table 3. Result of Ljung-Box Test

On standard. residuals		On Squared residuals	
Q(5)	40.48 (0.000)	$Q^2(5)$	767.51 (0.000)
Q(10)	54.14 (0.000)	$Q^2(10)$	1036.59 (0.000)
Q(20)	70.18 (0.000)	$Q^2(20)$	1352.06 (0.000)

Note: p values are in parenthesis

The diagnostic tests confirm the presence of conditional volatility in the rupee-dollar return series. Therefore, we proceed towards GARCH modelling. First we estimate an IGARCH(1,1) and an APARCH(1,1) model to see how the short memory models perform compared to the fractionally integrated models. The results are given in Table 4.

Table 4. Estimation results for IGARCH(1,1) and APARCH(1,1) models

Model	C	Α	β	Γ	Δ	ARCH (10, 4918)	Q2(20)	TIC
IGARCH(1,1) [normal]	0.00002 (0.000)	0.129 (0.000)	0.871 (0.000)			0.851 (0.577)	10.776 (0.903)	0.522
APARCH(1,1) [normal]	0.003 (0.094)	0.193 (0.000)	0.864 (0.000)	-0.210 (0.000)	1.071 (0.000	1.1780 (0.300)	13.619 (0.753)	0.524

Note: p values are in parenthesis

Here we can observe that the value of $\alpha+\beta\approx1$ in the IGARCH(1,1) model, signifying the persistence of volatility. Also, the asymmetry parameter in the APARCH(1,1) model is also significant, confirming that negative news can have more impact on market volatility than positive news.

Next, we analyse the presence of long memory in conditional volatility. This part of the analysis consists of two steps. In the first step we test for long memory in conditional volatility using ARFIMA(p,q)-FIGARCH(p,d,q) and ARFIMA(p,q)-FIAPARCH(p,d,q) models. In the second step we test for long memory in the conditional mean and the conditional variance using ARFIMA(p,d,q)-FIGARCH(p,d,q) and ARFIMA(p,d,q)-FIAPARCH(p,d,q) models. We assume normal, student-t, and skewed student-t distributional assumptions for both sets of estimations. Table 5 presents the parameter estimates obtained from ARFIMA(p,q)-FIGARCH(p,d,q) and ARFIMA(p,q)-FIAPARCH(p,d,q) models.

Table 5. Estimated Parameters of ARMA(1,2)-FIGARCH(1,d,1) and ARMA(1,2)-FIAPARCH(1,d,1)

Model	**AR(1)**	**MA(1)**	**MA(2)**	**C**	**D**	A	β	Γ	δ	ARCH F(10,4918)	$Q^2(20)$	TIC
FIGARCH (NORMAL)	-0.875 (0.000)	0.710 (0.000)	-0.161 (0.000)	0.002 (0.746)	0.446 (0.000)	0.388 (0.000)	0.627 (0.000)			0.655 (0.766)	9.084 (0.977)	0.491
FIGARCH (STD-T)	-0.743 (0.000)	0.574 (0.000)	-0.156 (0.000)	-0.005 (0.000)	0.713 (0.000)	0.241 (0.000)	0.658 (0.000)			0.121 (0.999)	2.536 (0.999)	0.521
FIGARCH (SK.STD-T)	-0.741 (0.000)	0.571 (0.000)	-0.156 (0.000)	-0.005 (0.000)	0.713 (0.000)	0.238 (0.000)	0.654 (0.000)			0.123 (0.999)	2.556 (0.999)	0.526
FIAPARCH (NORMAL)	0.944 (0.000)	-1.095 (0.000)	0.152 (0.000)	-0.002 (0.024)	0.478 (0.000)	0.301 (0.000)	0.558 (0.000)	-0.160 (0.010)	2.160 (0.000)	0.385 (0.954)	6.114 (0.953)	0.502
FIAPARCH (STD.T)	-0.742 (0.000)	0.579 (0.000)	-0.150 (0.000)	-0.006 (0.000)	0.636 (0.000)	0.265 (0.000)	0.628 (0.000)	-0.118 (0.002)	1.824 (0.000)	0.111 (0.999)	2.365 (0.999)	0.534
FIAPARCH (SK.STD-T)	-0.741 (0.000)	0.578 (0.000)	-0.149 (0.000)	-0.006 (0.000)	0.631 (0.000)	0.266 (0.000)	0.623 (0.000)	-0.111 (0.003)	1.816 (0.000)	0.110 (0.999)	2.366 (0.999)	0.531

Note: p values are in parenthesis

Here, the ARMA lags for the mean equation were selected using the AIC criterion. The use of the FIAPARCH model is justified as the asymmetry parameter is statistically significant and shows that negative news has more impact on market volatility than positive news.

Here, the results show a mixed picture as far as the presence of long memory in conditional volatility is concerned. While the FIGARCH model and the FIAPARCH model under the normal distribution assumption confirm the presence of long memory in conditional volatility ($d<0.5$), the other 4 models refute the presence of a long-term dependence structure. Hence, to pick up a particular model for practical purposes, we need to look for other measures that explain the model adequacy. The post-estimation diagnostic test results shows that all models successfully capture the volatility in the data. Therefore, we consider the in-sample forecast measures to compare the models with each other. From observing the values of Theil's inequality coefficient, it is clear that the FIGARCH(1,1) model [normal distribution] fares better than the others, followed by the FIAPARCH(1,1) model [normal distribution]. It should also be noticed that these models fare better than the IGARCH(1,1) and APARCH(1,1) models estimated in the previous step in terms of in-sample forecast ability.

In the next part of the analysis we test for long memory in both the conditional mean equation and the conditional variance equation. The results are displayed in Table 6.

Table 6. Estimated Parameters of ARFIMA(1,d,2)-FIGARCH(1,d,1) and ARFIMA(1,d,2)-FIAPARCH(1,d,1)

Model	d (ARFIMA)	AR(1)	MA(1)	MA(2)	C	D	α	Β	Γ	δ
FIGARCH (normal)	0.048 (0.034)	-0.858 (0.000)	0.642 (0.000)	-0.208 (0.000)	0.002 (0.731)	0.449 (0.000)	0.382 (0.000)	0.627 (0.000)		
FIGARCH (std-t)	0.056 (0.010)	-0.704 (0.000)	0.474 (0.000)	-0.203 (0.000)	0.000 (1.000)	0.730 (0.000)	0.225 (0.000)	0.662 (0.000)		
FIGARCH (sk.std-t)	0.066 (0.004)	-0.692 (0.000)	0.450 (0.000)	-0.210 (0.000)	0.000 (1.000)	0.729 (0.000)	0.221 (0.000)	0.658 (0.000)		
FIAPARCH (normal)	0.1698 (0.044)	0.807 (0.083)	-1.132 (0.000)	0.225 (0.021)	-0.001 (0.445)	0.482 (0.000)	0.295 (0.065)	0.557 (0.000)	-0.155 (0.013)	2.160 (0.000)
FIAPARCH (std.t)	0.061 (0.009)	-0.702 (0.000)	0.473 (0.000)	-0.200 (0.000)	-0.006 (0.000)	0.634 (0.000)	0.257 (0.000)	0.621 (0.000)	-0.135 (0.000)	1.828 (0.000)
FIAPARCH (sk.std.t)	0.073 (0.000)	-0.683 (0.000)	0.442 (0.000)	-0.207 (0.000)	0.000 (0.000)	0.645 (0.000)	0.245 (0.000)	0.622 (0.000)	-0.124 (0.000)	1.807 (0.000)

Note: p values are in parenthesis.

Here, we find evidence for long memory in conditional mean series for all the 6estimated models and thereby confirm the presence of long memory in the return series of the Indian rupee. However, as in the previous case, only the FIGARCH model and the FIAPARCH model under normal distribution assumption confirm the presence of long memory in conditional volatility. In such a scenario we resort to the analysis of in-sample forecast measures to pick up the adequate model(s). The results of post-estimation diagnostic tests and in-sample forecast measures are displayed in table 7.

Table 7. Post-estimation diagnostic test measures for ARFIMA(1,d,2)-FIGARCH(1,d,1) and ARFIMA(1,d,2)-FIGARCH(1,d,1)

Model	**ARCH F(10,4918)**	**$Q^2(20)$**	**TIC**
FIGARCH (normal)	0.704 (0.721)	9.578 (0.995)	0.491
FIGARCH (std-t)	0.156 (0.996)	2.967 (0.999)	0.591
FIGARCH (sk.std-t)	0.163 (0.999)	3.058 (0.999)	0.532
FIAPARCH (normal)	0.346 (0.968)	16.611 (0.999)	0.502
FIAPARCH (std.t)	0.146 (0.999)	2.798 (0.999)	0.542
FIAPARCH (sk.std.t)	0.150 (0.998)	8.998 (1.000)	0.537

Note: p values are in parenthesis

We can see that all the models adequately capture the volatility phenomenon. As in the previous case, the FIGARCH (1,1) model [normal distribution] gives the best in-sample forecast measure, followed by the FIAPARCH (1,1) model [normal distribution]. Here, too, the fractionally integrated models with long memory in both conditional mean and variance equations perform better than the short memory [IGARCH(1,1) & APARCH(1,1)] models.

Hence, it could be said that in order to accurately predict volatility in the Indian rupee, it is better to employ fractionally integrated GARCH models with long memory in both the conditional mean and variance equations.

5. CONCLUSION

In this study, we tried to analyse the presence of long memory in volatility in the Indian Forex market. We took the bilateral daily returns of the Indian rupee against the US dollar from 17/02/1994 to 08/11/2013 for the purpose of the analysis. In the first part we tested for long memory in the return series, the absolute return series, and the squared return series, the latter two serving as a measure of unconditional volatility.

Next, we analysed the presence of long memory in conditional volatility by employing an AR(FI)MA(1,2)-FIGARCH(1,1) model and an AR(FI)MA(1,2)-FIAPARCH(1,1) model. The results gave a mixed picture, where the ARMA(1,2)-FIGARCH (1,1) model [normal distribution] and the ARMA(1,2)-FIAPARCH (1,1) model [normal distribution] confirmed the presence of long memory in conditional variance, while the other models refuted it. In terms of in-sample forecast accuracy, these two models performed better than the short-memory models that we estimated for comparison purposes.

While testing for long memory in both conditional mean and variance, the ARFIMA(1,d,2)-FIGARCH (1,1) model [normal distribution] and the ARFIMA(1,d,2)-FIAPARCH (1,1) model [normal distribution] confirmed the presence of long memory in both conditional mean and variance, while the other models only confirmed the presence of long memory in conditional mean.

To pick an adequate model, we compared them on the basis of in-sample forecast measures against the two estimated short-memory models. The in-sample forecast measures indicated that in order to capture the market volatility it wouldbe better to use fractionally integrated models than short-memory models. It was found that the Indian Forex market has an underlying fractal structure, as the presence of long memory was confirmed by the analysis.

The presence of long memory can have multiple implications. From a theoretical point of view the presence of self-similar patterns refutes the notion of the efficient market hypothesis in its weak form. Further,the presence of long memory in returns and volatility series indicates that it would be better to develop and employ long-memory models as opposed to the traditional GARCH models to forecast market returns and volatility.

From an investors' perspective, the presence of patterns in the market in both returns and volatility structure implies that it will be possible to predict trends in returns and volatility. Hence the possibility of gaining extra-normal profit and diversifying risk exists.

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